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CHAPTER

7 Linear Equations and Inequalities in Two Variables

A company that manufactures a heating unit can produce 20 units for \$13,900 while it would cost \$7,500 to manufacture 10 units. Assume the cost and number of units produced are related by the linear equation of a straight line. Let y be the total cost to manufacture x units. Find the linear equation of the straight line.

7–1 ■ The rectangular coordinate system

In chapter 2, we considered the solution set of linear equations (first-degree equations) in one variable. That is, equations of the form

$$ax + b = 0$$

where a and b are real numbers, $a \neq 0$. The solution sets of these equations are sets of real numbers.

In this chapter, we expand our work with equations to consider **linear equations in two variables,** x and y. Such equations are of the form

$$ax + by = c$$

where a, b, and c are real numbers, not both a and b equal to zero. The equations

$$3x + y = 4, \quad 4y - x = 0, \quad y = 2x - 1, \quad \text{and} \quad x = y - 4$$

are examples of linear equations in two variables.

In an equation in two variables, x and y, the x and y are replaced by a pair of numbers. If that pair of numbers makes the equation true, we say the pair of numbers satisfies the equation. Any pair of numbers that satisfies the equation is a solution of that equation. Consider the linear equation $3x - y = 5$. Let $x = 2$ and $y = 1$. If we substitute 2 for x and 1 for y in the equation, we have

$$\begin{aligned} 3(2) - (1) &= 5 \\ 6 - 1 &= 5 \\ 5 &= 5 \quad \text{(True)} \end{aligned}$$

Therefore the values 2 for x and 1 for y form a solution of the equation $3x - y = 5$. The pair of numbers $x = 2$ and $y = 1$ that form this solution are usually written in the form $(2,1)$. This pair of numbers is called an **ordered pair of real numbers** because the numbers are written in a specific order, x first and then y, (x,y). We call x the **first component** and y the **second component** of the ordered pair (x,y).

To graph an ordered pair, we use two real number lines that intersect at right angles with each other at their zero points. The point of intersection is called the **origin.** The two lines, one *horizontal* and the other *vertical,* are called **axes.** The horizontal line, called the **x-axis,** is associated with the first number of the ordered pair, and the vertical line, called the **y-axis,** is associated with the second number of the ordered pair. The x-axis and the y-axis form the **rectangular coordinate system** and partition the plane into four equal regions called **quadrants.** The quadrants are numbered I, II, III, and IV in a counterclockwise direction. See figure 7–1.

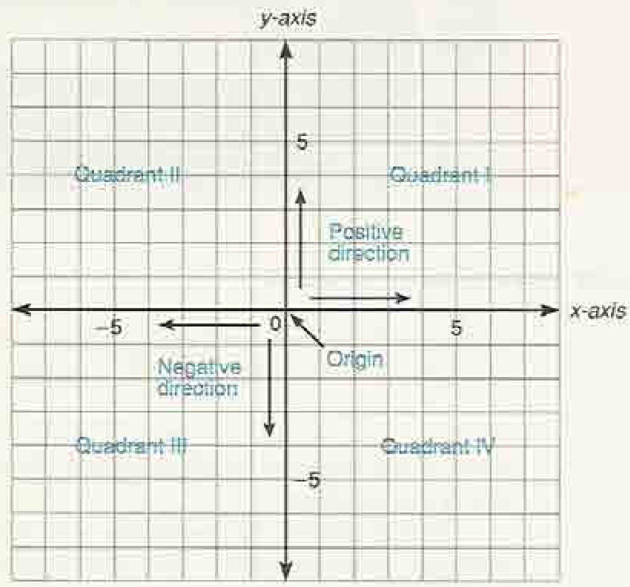


Figure 7–1

Plotting Ordered Pairs

To locate the point in the plane that corresponds to the ordered pair $(3,5)$, we start at the origin and move 3 units to the *right* (the positive direction) along the (horizontal) x-axis, and then we move 5 units *up* (the positive direction) along the y-axis (vertical). See figure 7–2(a). To locate the point in the plane that corresponds to the ordered pair $(-5,-4)$, we start at the origin and move 5 units to the *left* (the negative direction) along the x-axis (horizontal) and then we move 4 units *down* (the negative direction) along the y-axis (vertical). See figure 7–2(b).

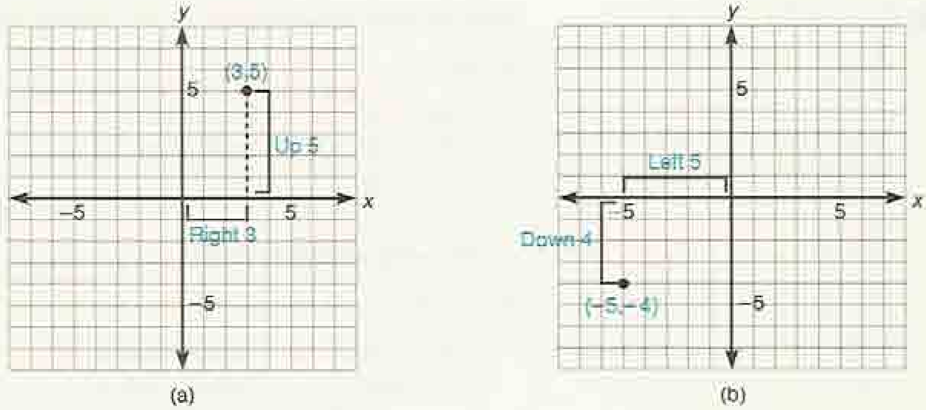


Figure 7–2

In the ordered pair $(-5,-4)$, we call the numbers -5 and -4 the **coordinates of the point,** where the first number, -5, is called the **abscissa,** or x-coordinate, of the point and the second number, -4, is called the **ordinate,** or y-coordinate, of the point.

Note The coordinates of the origin are (0,0).

Points are usually named by capital letters and/or their coordinates. When we use the notation $P(x,y)$, we mean the point P whose coordinates are x and y. For example, in figure 7–3 the points $A(4,4)$, $B(0,3)$, $C(-4,2)$, $D(-3,0)$, $E(-6,-2)$, $F(0,-5)$, $G(2,-3)$, $H(7,0)$, and $I(0,0)$ have been located in the plane. Point A lies in quadrant I, point C lies in quadrant II, point E lies in quadrant III, and point G lies in quadrant IV.

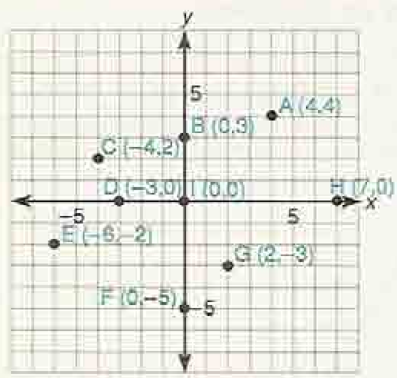


Figure 7–3

To find ordered pairs that are solutions of a linear equation in two variables, we use the following procedure.

Finding ordered pair solutions

1. Choose any real number value of one of the variables (usually x).
2. Replace that variable with the chosen value and solve the resulting equation in one variable.
3. Write the solution as an ordered pair (x,y).

■ **Example 7–1 A**

Find the missing component of the ordered pair for the equation $3x + 2y = 6$.

1. (0,) $x = 0$

$$\begin{aligned} 3x + 2y &= 6 && \text{Original equation} \\ 3(0) + 2y &= 6 && \text{Replace } x \text{ with } 0 \\ 2y &= 6 && \text{Solve for } y \\ y &= 3 \end{aligned}$$

The ordered pair is (0,3).

2. (,0) $y = 0$

$$\begin{aligned} 3x + 2y &= 6 && \text{Original equation} \\ 3x + 2(0) &= 6 && \text{Replace } y \text{ with } 0 \\ 3x &= 6 && \text{Solve for } x \\ x &= 2 \end{aligned}$$

The ordered pair is (2,0).

3. (4,) $x = 4$

$$\begin{aligned} 3x + 2y &= 6 && \text{Original equation} \\ 3(4) + 2y &= 6 && \text{Replace } x \text{ with } 4 \\ 12 + 2y &= 6 && \text{Solve for } y \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

The ordered pair is $(4,-3)$.

▶ ***Quick check*** Find the missing components for the ordered pairs for the equation $4x - 3y = 12$; (0,), (,0), (6,). ■

Graphing an equation

To graph the equation $3x + 2y = 6$, we could first graph the ordered pairs (0,3), (2,0), and $(4,-3)$ found in example 7–1 A. The points appear to lie on a straight line. In fact, if *all* ordered pair solutions of $3x + 2y = 6$ were plotted, the points would lie on this straight line. See figure 7–4 for the graph of $3x + 2y = 6$.

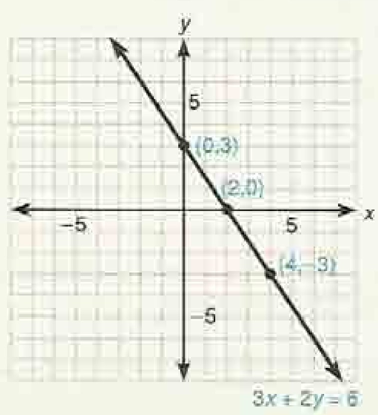


Figure 7–4

The graph of a linear equation of the form

$$ax + by = c \text{ (} a \text{ and } b \text{ not both 0)}$$

is a straight line.

Since a straight line is determined by any two distinct points on the line, finding two points on the line is sufficient to graph the equation. Two points most easily found are the

1. x-intercept—the point (if any exists) where the graph crosses the x-axis. This occurs when $y = 0$.
2. y-intercept—the point (if any exists) where the graph crosses the y-axis. This occurs when $x = 0$.

To guard against arithmetic error, it is wise to find a third point to act as a checkpoint.

Graphing a linear equation in two variables

1. Let $y = 0$ and solve for x to find the x-intercept, the point $(x,0)$.
2. Let $x = 0$ and solve for y to find the y-intercept, the point $(0,y)$.
3. Find a third point as a checkpoint.

Example 7–1 B

Find the x- and y-intercepts and sketch the graph of each equation.

1. $3x + 5y = 15$

a. Let $y = 0$, then

$$\begin{aligned} 3x + 5(0) &= 15 && \text{Replace } y \text{ with } 0 \\ 3x + 0 &= 15 && \text{Solve for } x \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

The x-intercept is the point $(5,0)$.

b. Let $x = 0$, then

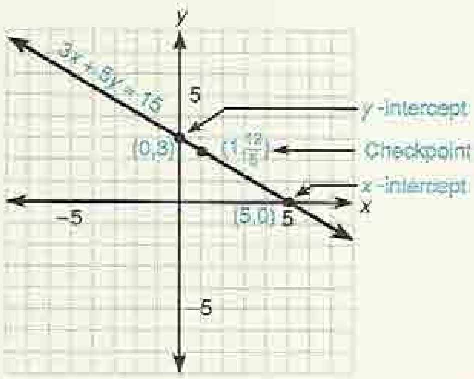
$$\begin{aligned} 3(0) + 5y &= 15 && \text{Replace } x \text{ with } 0 \\ 0 + 5y &= 15 && \text{Solve for } y \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

The y-intercept is the point $(0,3)$.

c. Checkpoint: Let $x = 1$.

$$\begin{aligned} 3(1) + 5y &= 15 && \text{Replace } x \text{ with } 1 \\ 3 + 5y &= 15 && \text{Solve for } y \\ 5y &= 12 \\ y &= \frac{12}{5} \end{aligned}$$

The checkpoint is $\left(1,\frac{12}{5}\right)$.



Note In future examples, we will not always show the checkpoint; however we should *always* find a third point as a check.

2. $4x = y$

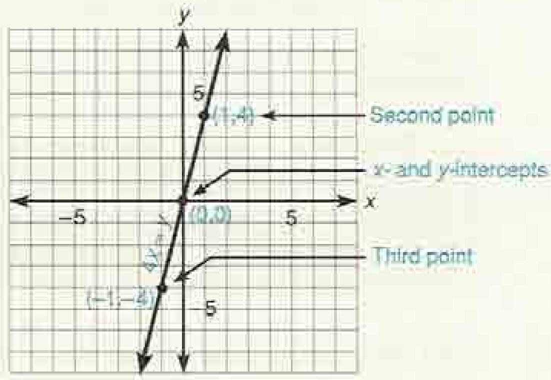
Let $x = 0$, then $4(0) = y$ Replace x with 0

$y = 0$

The y-intercept (and the x-intercept) is the point (0,0), which is the origin. Since both the intercepts are the same point, we must find two more distinct points.

Let $x = 1$	Let $x = -1$	
$4(1) = y$	$4(-1) = y$	Replace x with 1 and -1
$4 = y$	$-4 = y$	

Two additional points on the line are (1,4), and $(-1,-4)$.



3. $y = -3$

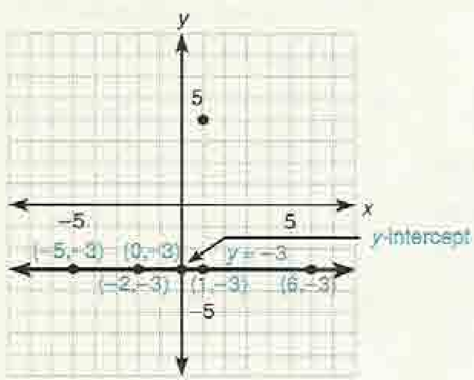
We can write this equation as

$0 \cdot x + y = -3$

and for any value of x that we might choose, y is *always* -3. That is,

$(-5,-3)$, $(-2,-3)$, $(0,-3)$, $(1,-3)$, and $(6,-3)$

are all solutions of the equation. Plotting these points and drawing a straight line through them, the graph is a horizontal line (parallel to the x-axis) having a y-intercept of $(0,-3)$ and no x-intercept.

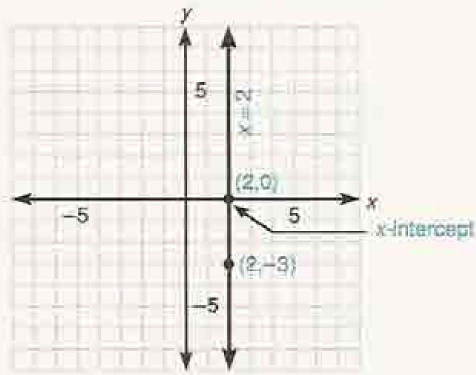


4. $x = 2$

We can write the equation as

$x + 0 \cdot y = 2$

and for any value of y that we choose, x is always 2. If we choose two solutions, say $(2,-3)$ and $(2,0)$, and draw a straight line through these points, we have the graph of $x = 2$. The graph is a vertical line (parallel to the y-axis) having an x-intercept of $(2,0)$ and no y-intercept.



▶ ***Quick check*** Find the x- and y-intercepts and sketch the graph for $2x + 7y = 14$.

In general, from examples 3 and 4, we see that the following is true:

If k is some real number, $k \neq 0$, the graph of

1. $x = k$ is a vertical line with an x-intercept $(k,0)$ and no y-intercept.
2. $y = k$ is a horizontal line with a y-intercept $(0,k)$ and no x-intercept.

Mastery points

Can you

- Plot the graph of an ordered pair of real numbers?
- Determine in what quadrant a point lies?
- Find the x- and y-intercepts of a linear equation in two variables?
- Sketch the graph of a linear equation in two variables?
- Graph equations $x = k$ and $y = k$, k is a constant?

Exercise 7–1

Plot the graph of the following ordered pairs of real numbers. State the quadrant in which the point lies.

1. $(2,4)$ **2.** $(5,2)$ **3.** $(-4,3)$ **4.** $(-6,5)$ **5.** $(-1,-3)$

6. $(-4,-1)$ **7.** $(4,0)$ **8.** $(-6,0)$ **9.** $(0,-1)$ **10.** $(0,5)$

11. $\left(\frac{1}{2},3\right)$ **12.** $\left(-2,\frac{3}{2}\right)$ **13.** $\left(-\frac{7}{2},-\frac{5}{2}\right)$

For each equation, find the missing value in the ordered pairs. Sketch the graph of the equation using these ordered pairs. See example 7–1 A.

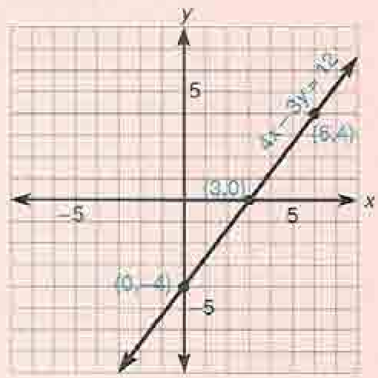
Example $4x - 3y = 12$; (0,), (,0), (6,)

Solution a. (0,)

$$4(0) - 3y = 12 \quad \text{Replace } x \text{ with } 0$$
$$-3y = 12 \quad \text{Solve for } y$$
$$y = -4 \quad \text{Ordered pair } (0,-4)$$

b. (,0)

$$4x - 3(0) = 12 \quad \text{Replace } y \text{ with } 0$$
$$4x - 0 = 12 \quad \text{Solve for } x$$
$$4x = 12 \quad \text{Ordered pair } (3,0)$$
$$x = 3$$



c. (6,)

$$4(6) - 3y = 12 \quad \text{Replace } x \text{ with } 6$$
$$24 - 3y = 12 \quad \text{Solve for } y$$
$$-3y = -12$$
$$y = 4 \quad \text{Ordered pair } (6,4)$$

14. $3x + y = -1$; $(-3,\ )$, $(-1,\ )$, $(0,\ )$, $(\ ,0)$

15. $2x + y = 3$; $(-2,\ )$, $(0,\ )$, $(2,\ )$, $(\ ,0)$

16. $x - y = 2$; $(-2,\ )$, $(0,\ )$, $(2,\ )$, $(\ ,0)$

17. $x + y = 4$; $(-5,\ )$, $(-3,\ )$, $(0,\ )$, $(\ ,0)$

18. $x + 2y = 4$; $(-2,\ )$, $(0,\ )$, $(2,\ )$, $(\ ,0)$

19. $y - x = -2$; $(-3,\ )$, $(0,\ )$, $(2,\ )$, $(\ ,0)$

20. $2x + 5y = 20$; $(-5,\ )$, $(0,\ )$, $(5,\ )$, $(\ ,0)$

21. $x - 3y = 1$; $(-1,\ )$, $(0,\ )$, $(1,\ )$, $(\ ,0)$

22. $4x - 3y = 6$; $(-6,\ )$, $(-3,\ )$, $(0,\ )$, $(\ ,0)$

23. $3x + 2y = 8$; $(-2,\ )$, $(0,\ )$, $(2,\ )$, $(\ ,0)$

Plot the x- and y-intercepts for the graph of each equation, if they exist. Sketch the lines. See example 7–1 B.

Example $2x + 7y = 14$

Solution a. Let $x = 0$, then

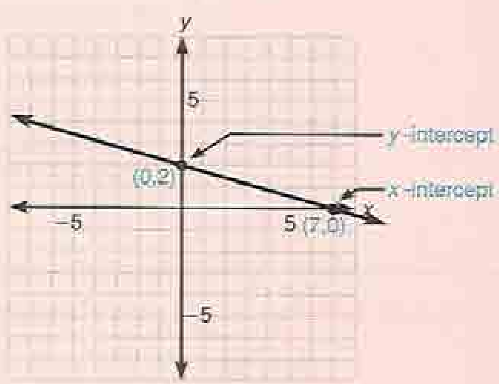
$$2(0) + 7y = 14 \quad \text{Replace } x \text{ with } 0$$
$$7y = 14 \quad \text{Solve for } y$$
$$y = 2$$

y-intercept is (0,2).

b. Let $y = 0$, then

$$2x + 7(0) = 14 \quad \text{Replace } y \text{ with } 0$$
$$2x = 14 \quad \text{Solve for } x$$
$$x = 7$$

x-intercept is (7,0).



24. $x + 2y = 4$

25. $x - 3y = -6$

26. $4x - 5y = 20$

27. $5x + 2y = 20$

28. $4x + y = 8$

29. $5x - y = -10$

30. $x = 3y$

31. $x = -2y$

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32. $y - 3x = 0$
33. $y + 2x = 0$
34. $4y - x = 0$
35. $5y - 3x = 6$
36. $3x + 2y = 8$
37. $y = -2$
38. $y = 6$
39. $x = -1$
40. $x = 8$
41. $x = 0$
42. $y = 0$

Translate each of the following statements into an equation and graph the equation.

43. If 4 is added to x, the result is y.
44. Two times the value of x less 3 is equal to y.
45. Three times x less two times y is 12.
46. If 3 is added to the y-value, the result is four times the x-value.
47. Temperature measured in Fahrenheit degrees can be converted to Celsius degrees using the equation

$$C = \frac{5}{9}(F - 32)$$

Let the horizontal axis represent F and the vertical axis represent C. Graph the equation.

48. Graph the lines $y = x + 2$ and $y = x - 1$ on the same rectangular coordinate system. What appears to be true of the lines? From what you observe, the graph of $y = x + 6$ will be where in the plane?
49. Graph the lines $y = x + 2$ and $y = 2x - 3$ on the same rectangular coordinate system. At what point do the lines appear to cross?

Review exercises

Perform the indicated operations. Write your answer in the standard form $a + bi$. See section 5–7.

1. $(2 + 3i)(4 - i)$
2. $(4 + i)(4 - i)$
3. $(2 + \sqrt{-9})(2 - 2\sqrt{-9})$
4. $\dfrac{3 + i}{3 - i}$
5. Solve the formula $P = 2\ell + 2w$ for w. See section 2–2.
6. Divide $(x^4 - 1)$ by $(x - 1)$. See section 4–6.
7. Evaluate $\dfrac{p - q}{r - s}$ when $p = 2$, $q = 4$, $r = -3$, and $s = -5$. See section 1–5.

7–2 ■ The distance formula and the slope of a line

Distance formula

In section 7–1, we studied the graph of a linear equation that is a straight line. If we choose any two points on that line, the portion of the line between the two points is called a **line segment.** A line has no length, while a line segment has a specific length. We cannot determine the length of a line since it continues indefinitely in both directions, but we can determine the length of a line segment. The length of the line segment is defined as the **distance** between the two points. See figure 7–5.

Figure 7–5

Given line L containing points P_1 and P_2, the length of the line segment from P_1 to P_2 is then called the *distance* from P_1 to P_2. Let us consider three specific examples and then develop the distance formula.

Consider three points in the plane, (3,2), (3,−3), and (−2,−3), shown in figure 7–6.

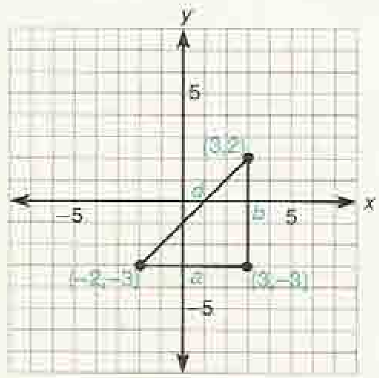


Figure 7–6

The points form a right triangle having sides a, b, and d. The length of

1. side a, which is parallel to the x-axis is
$$|-2-3| = |3-(-2)| = 5 \text{ units long}$$
2. side b, which is parallel to the y-axis, is
$$|2-(-3)| = |-3-2| = 5 \text{ units long}$$

To find the length of side d, we use the Pythagorean Theorem mentioned in chapters 5 and 6 which states

$$d^2 = a^2 + b^2$$

Thus,
$$\begin{aligned} d^2 &= 5^2 + 5^2 \\ &= 25 + 25 \\ &= 50 \end{aligned}$$

Extracting the roots,
$$d = \pm\sqrt{25} = \pm 5\sqrt{2}$$

Since distance is nonnegative, the distance $d = 5\sqrt{2}$ units.

Now consider the distance between arbitrary points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, denoted by $d(P_1P_2)$. See figure 7–7.

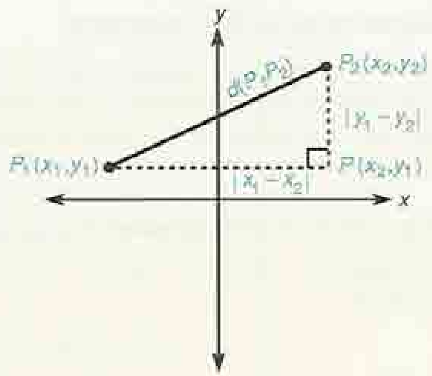


Figure 7–7

We have drawn a horizontal dashed line segment from P_1 and a vertical dashed line segment from P_2 so that these segments meet at point $P(x_2,y_1)$, thus forming a right triangle. The lengths of the dashed line segments by definition are $d(P_1P) = |x_2 - x_1|$ and $d(P_2P) = |y_2 - y_1|$. To find the distance from P_1 to P_2, denoted by d, we use the **Pythagorean Theorem.**

Since the square of any number is never negative, then

$$|x_2 - x_1|^2 = (x_2 - x_1)^2 \quad \text{and} \quad |y_2 - y_1|^2 = (y_2 - y_1)^2$$

and so

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Distance is never negative, so we use the principal, or positive, square root to find d.

Distance formula

The distance between any two points (x_1,y_1) and (x_2,y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note When working with two points on a line, it doesn't make any difference which point is labeled (x_1,y_1) and which point is labeled (x_2,y_2).

Example 7–2 A

Find the distance d from (3,2) to (6,6).

Let $(x_1,y_1) = (3,2)$ and $(x_2,y_2) = (6,6)$.

$$\begin{aligned} d &= \sqrt{(6-3)^2 + (6-2)^2} \quad \text{Replace } x_1 \text{ with 6, } x_2 \text{ with 3, } y_1 \text{ with 6, and } y_2 \text{ with 2} \\ &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} = 5 \end{aligned}$$

Thus $d = 5$ units.

▶ ***Quick check*** Find the distance d from $(-4,3)$ to $(5,-6)$.

The midpoint of a line segment

Sometimes we must find the midpoint of a line segment. We now give the formula for finding the coordinates of this point. Using similar triangles, it can be shown that the coordinates of the point midway between two given points are found by averaging the x-coordinates and the y-coordinates of the points.

Midpoint of a line segment

The **midpoint** of the line segment joining points (x_1,y_1) and (x_2,y_2) has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 7–2 B

Find the midpoint of the line segment whose endpoints are $(6,-4)$ and $(4,2)$.
Let $(x_1,y_1) = (6,-4)$ and $(x_2,y_2) = (4,2)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(6) + (4)}{2}, \frac{(-4) + (2)}{2}\right)$$

Replace x_1 with 6, x_2 with 4, y_1 with -4, and y_2 with 2

$$= \left(\frac{10}{2}, \frac{-2}{2}\right)$$
$$= (5,-1)$$

The midpoint of the line segment is the point $(5,-1)$.

▶ *Quick check* Find the midpoint of the line segment whose endpoints are $(-4,3)$ and $(5,-6)$.

The slope of a line

Now consider the portions of two lines as one moves from point P_1 to point P_2 on each incline. See figure 7–8.

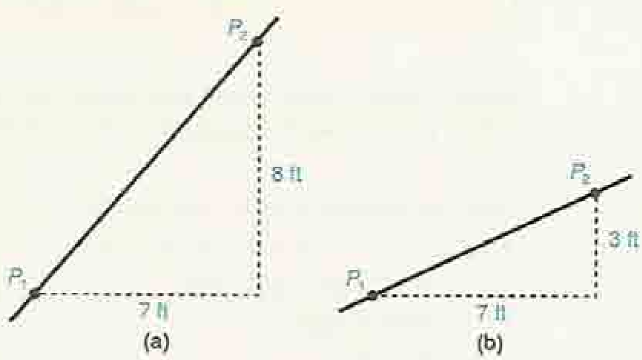


Figure 7–8

The incline in figure 7–8(a) is "steeper" than the incline in figure 7–8(b). That is, the inclination in figure 7–8(a) is greater than the inclination in figure 7–8(b). In moving from point P_1 to P_2, the horizontal change is 7 feet in both cases, but the vertical change in (a) is 8 feet and the vertical change in (b) is 3 feet. If we measure this "steepness," or inclination, by the quotient

$$\text{steepness} = \frac{\text{vertical change}}{\text{horizontal change}}$$

the "steepness" of the line in

$$\text{(a) is } \frac{8 \text{ feet}}{7 \text{ feet}} = \frac{8}{7}$$

and of the line in

$$\text{(b) is } \frac{3 \text{ feet}}{7 \text{ feet}} = \frac{3}{7}$$

Note $\frac{8}{7}$ is greater than $\frac{3}{7}$, so the line in (a) is "steeper" than the line in (b).

When applying this concept to any straight line, this "steepness" is called the **slope** of the line. Therefore the slope of a line is given by

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

See figure 7–9.

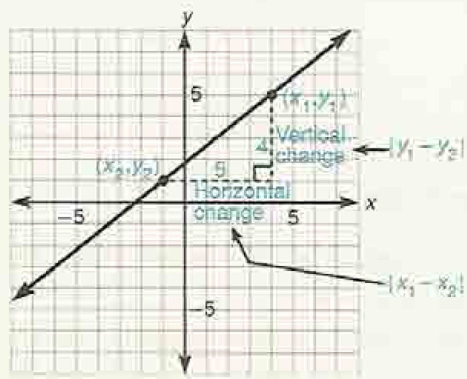


Figure 7–9

The vertical change is 4 units and the horizontal change is 5 units. Then the slope of the line is given by

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{4}{5}$$

Note This means that for every 5 units moved to the right from a point on the line, there must be a rise of 4 units to get back to the line.

We denote the slope of a line by m.

Definition of the slope of a straight line

If $(x_1 \neq x_2)$, the slope (m) of the line containing points (x_1,y_1) and (x_2,y_2) is defined by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Concept

The slope of a line is obtained by dividing the change in y-values by the corresponding change in x-values.

Note It is a common mistake to write $m = \frac{y_1 - y_2}{x_2 - x_1}$ or $m = \frac{y_2 - y_1}{x_1 - x_2}$. The order in which the x-values are subtracted *must be the same* as the order in which the y-values are subtracted.

The slope of a line can alternately be defined by

$$m = \frac{\text{rise}}{\text{run}}$$

where the vertical change is the **rise** and the horizontal change is the **run.** The slope m is the amount of rise or fall for each unit of run. See figure 7.10.

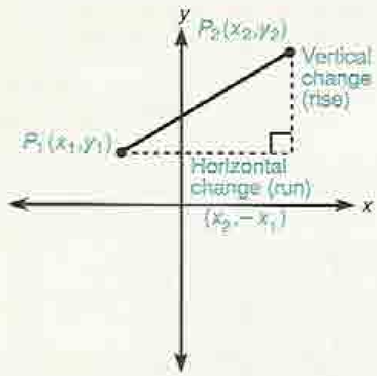


Figure 7–10

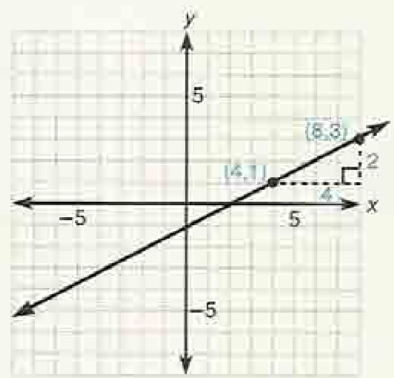
■ Example 7–2 C

Find the slope of the line passing through the given points. Sketch the line.

1. (4,1) and (8,3)
   Let $(x_1,y_1) = (4,1)$ and $(x_2,y_2) = (8,3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3) - (1)}{(8) - (4)} \quad \text{Replace } y_2 \text{ with 3, } y_1 \text{ with 1, } x_2 \text{ with 8, and } x_1 \text{ with 4}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

The slope of the line is $\frac{1}{2}$.



Note The x- and y-values may be subtracted in any order so long as the coordinates of each point are in the same position in the numerator and the denominator. That is, we could use

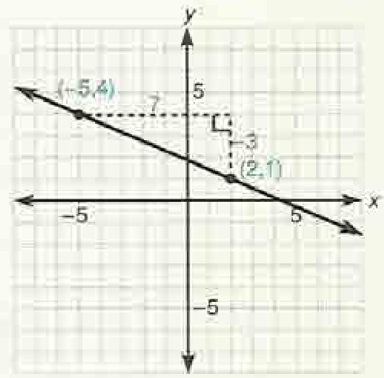
$$m = \frac{y_1 - y_2}{x_1 - x_2}$$
$$= \frac{(1) - (3)}{(4) - (8)} = \frac{-2}{-4} = \frac{1}{2}$$

2. $(-5,4)$ and $(2,1)$
Let $(x_1,y_1) = (-5,4)$ and $(x_2,y_2) = (2,1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(1) - (4)}{(2) - (-5)} = \frac{-3}{7} = -\frac{3}{7}$$

Replace y_2 with 1, y_1 with 4, x_2 with 2, and x_1 with -5

Thus the slope of the line is $-\frac{3}{7}$.



Note For every 7 units moved to the right from a point on the line, there must be a "fall" of 3 units to get back to the line.

We see that the graph of a line having a positive slope (example 1) "slants" *up* from left to right, and the graph of a line having negative slope (example 2) "slants" *down* from left to right. This will always be the case.

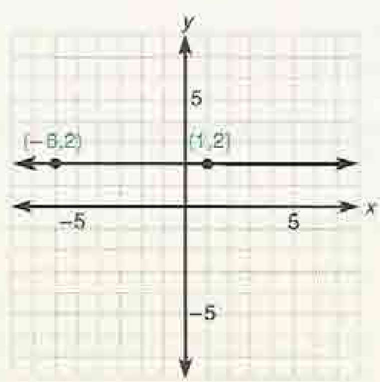
3. $(-6,2)$ and $(1,2)$
Let $(x_1,y_1) = (-6,2)$ and $(x_2,y_2) = (1,2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (2)}{(1) - (-6)} = \frac{0}{7} = 0$$

Replace y_2 with 2, y_1 with 2, x_2 with 1, and x_1 with -6

Thus the slope of the line is 0.

The points lie on a horizontal line, and the slope $m = 0$.



Slope of a horizontal line

The slope m of any horizontal line is 0.

4. $(-3,-1)$ and $(-3,4)$
   Let $(x_1,y_1) = (-3,-1)$ and $(x_2,y_2) = (-3,4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{-3 - (-3)}$$

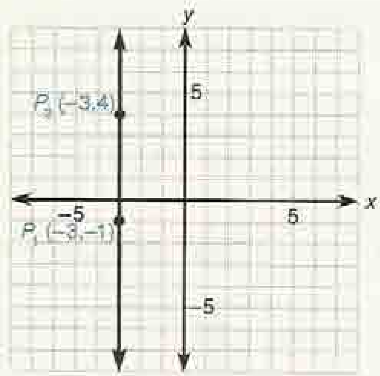
Replace y_2 with 4, y_1 with -1, x_2 with -3, and x_1 with -3

$$= \frac{5}{-3 + 3}$$

$$= \frac{5}{0} = \text{undefined}$$

The slope of the line is undefined.

Note In our definition of slope, we placed the restriction $x_1 \neq x_2$. In this example, $x_1 = x_2 = -3$.



The points lie on a vertical line, and the slope is undefined.

Slope of a vertical line

The slope m of a vertical line is undefined.

▶ ***Quick check*** Find the slope of the line passing through $(-4,3)$ and $(5,-6)$. Sketch the line.

On the basis of the discussion and examples, we can summarize the slope of a line as follows.

The slope of a line is

1. positive if the line slants up from left to right.
2. negative if the line slants down from left to right.
3. zero if the line is horizontal (parallel to the x-axis).
4. undefined if the line is vertical (parallel to the y-axis).

Parallel lines

Parallel lines are defined to be straight lines in the same plane that never meet. For this to be the case, the lines must have the same slope. See figure 7–11.

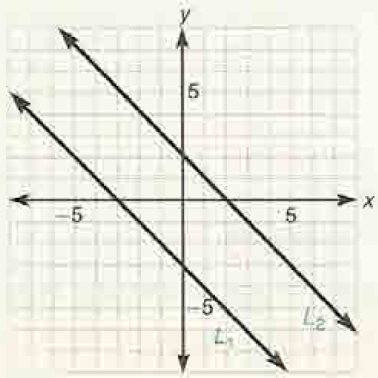


Figure 7–11

Definition of parallel lines

Given line L_1 has slope m_1 and line L_2 has slope m_2,

1. L_1 is parallel to L_2 if $m_1 = m_2$.
2. $m_1 = m_2$ if L_1 is parallel to L_2.

Concept

Two nonvertical lines are parallel if and only if their slopes are the same.

Note *All* vertical lines are parallel even though the slope of a vertical line is undefined.

Example 7–2 D

Determine if the line containing the points $(-3,2)$ and $(5,1)$ is parallel to the line containing the points $(1,5)$ and $(-7,6)$.

Using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$,

a. Let $(x_1,y_1) = (-3,2)$ and $(x_2,y_2) = (5,1)$.

$$m_1 = \frac{1-2}{5-(-3)} = \frac{-1}{8} = -\frac{1}{8}$$

b. Let $(x_1,y_1) = (1,5)$ and $(x_2,y_2) = (-7,6)$.

$$m_2 = \frac{6-5}{-7-1} = \frac{1}{-8} = -\frac{1}{8}$$

Both the slopes are $-\dfrac{1}{8}$ so the lines are parallel.

▶ ***Quick check*** Determine if the line containing the points $(1,2)$ and $(-3,6)$ is parallel to the line containing the points $(5,-6)$ and $(-2,1)$.

Perpendicular lines

It can be shown that the *product* of the slopes of two nonvertical perpendicular lines (lines that form right angles) is -1. For example, two lines whose slopes are $\frac{2}{3}$ and $-\frac{3}{2}$, respectively, are perpendicular since

$$\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$$

Notice that $\frac{2}{3}$ and $-\frac{3}{2}$ are *negative reciprocals* of one another. This gives rise to the following definition.

Definition of perpendicular lines

Given line L_1 has slope m_1 and line L_2 has slope m_2,

1. L_1 is perpendicular to L_2 if $m_1 \cdot m_2 = -1$.
2. $m_1 \cdot m_2 = -1$ if L_1 is perpendicular to L_2.

Concept

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1; that is, provided their slopes are negative reciprocals.

See figure 7–12.

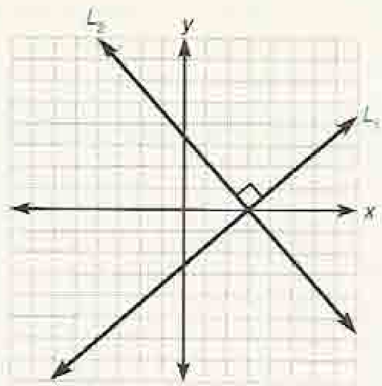


Figure 7–12

Note Any horizontal line is perpendicular to any vertical line.

Example 7–2 E

1. Determine if the line containing points (1,5) and $(-2,3)$ is perpendicular to the line containing the points $(5,-3)$ and (3,0).

   Using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$,

   a. Let $(x_1,y_1) = (1,5)$ and $(x_2,y_2) = (-2,3)$.

   $$m_1 = \frac{3-5}{-2-1} = \frac{-2}{-3} = \frac{2}{3}$$

b. Let $(x_1,y_1) = (5,-3)$ and $(x_2,y_2) = (3,0)$.

$$m_2 = \frac{0-(-3)}{3-5} = \frac{3}{-2} = -\frac{3}{2}$$

Since $\frac{2}{3}\left(-\frac{3}{2}\right) = -1$, the slopes are negative reciprocals and the lines are perpendicular.

2. Determine if the graphs of the equations $2x - 4y = 4$ and $4x + 2y = -5$ are perpendicular.

If we choose two ordered pairs that are in the solution set of each equation, we can determine the slopes of each line. Find the intercepts of each graph.

a. For $2x - 4y = 4$, the x-intercept is 2 and the y-intercept is -1. Then $(2,0)$ and $(0,-1)$ are solutions and

$$m_1 = \frac{0-(-1)}{2-0} = \frac{1}{2}$$

b. For $4x + 2y = -5$, the x-intercept is $-\frac{5}{4}$ and the y-intercept is $-\frac{5}{2}$. Therefore $\left(-\frac{5}{4},0\right)$ and $\left(0,-\frac{5}{2}\right)$ are solutions and

$$m_2 = \frac{0-\left(-\frac{5}{2}\right)}{-\frac{5}{4}-0} = \frac{\frac{5}{2}}{-\frac{5}{4}} = \frac{5}{2}\cdot\left(-\frac{4}{5}\right) = -2$$

Thus the graphs of the equations $2x - 4y = 4$ and $4x + 2y = -5$ are perpendicular lines since $m_1m_2 = \frac{1}{2}(-2) = -1$ and the slopes are negative reciprocals.

▶ ***Quick check*** Determine if the line containing the points $(1,-2)$ and $(3,5)$ is perpendicular to the line containing the points $(-1,-1)$ and $(6,-3)$. ■

Mastery points

Can you

- Find the distance between two points in the rectangular coordinate plane?
- Find the coordinates of the midpoint of a line segment?
- Find the slope of a straight line given two points on the line?
- Determine if two lines are parallel?
- Determine if two lines are perpendicular?

Exercise 7–2

In exercises 1–15, find the distance between the given pairs of points and the slope of the line containing the points. Find the midpoint in exercises 1–10. See examples 7–2 A, B, and C.

Example Let $(x_1,y_1) = (-4,3)$ and $(x_2,y_2) = (5,-6)$.

Solution a. Using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$,

$$\begin{aligned} d &= \sqrt{[(5) - (-4)]^2 + [(-6) - (3)]^2} \\ &= \sqrt{(9)^2 + (-9)^2} \\ &= \sqrt{81 + 81} = \sqrt{162} = \sqrt{81 \cdot 2} = 9\sqrt{2} \end{aligned}$$

Replace x_1 with -4, x_2 with 5, y_1 with 3, and y_2 with -6

The distance between the points is $9\sqrt{2}$ units.

b. Using $m = \dfrac{y_1 - y_2}{x_1 - x_2}$,

$$\begin{aligned} m &= \frac{(3) - (-6)}{(-4) - (5)} \\ &= \frac{9}{-9} = -1 \end{aligned}$$

Replace x_1 with -4, x_2 with 5, y_1 with 3, and y_2 with -6

The slope is -1.

c. Using $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$,

the midpoint is
$$\begin{aligned} &\left(\frac{(-4) + (5)}{2}, \frac{(3) + (-6)}{2}\right) \\ &= \left(\frac{1}{2}, \frac{-3}{2}\right) \\ &= \left(\frac{1}{2}, -\frac{3}{2}\right) \end{aligned}$$

Replace x_1 with -4, x_2 with 5, y_1 with 3, and y_2 with -6

The midpoint of the line segment is $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

1. (2,2) and (6,7)

2. (4,1) and (1,4)

3. $(-1,5)$ and $(-3,0)$

4. $(-4,6)$ and $(-1,2)$

5. $(-1,4)$ and $(-1,9)$

6. $(2,-6)$ and (2,1)

$\boxed{7.}$ (3,6) and $(-4,6)$

8. $(-1,-3)$ and $(5,-3)$

9. $(-3,-1)$ and $(-4,0)$

10. (3,5) and $(-2,6)$

11. $(7,-3)$ and $(-4,4)$

12. $(-2,-2)$ and $(6,-3)$

13. $(4,-5)$ and (2,4)

$\boxed{14.}$ (0,8) and $(0,-1)$

15. $(0,-6)$ and (0,4)

Determine if lines L_1 and L_2 are parallel. See example 7–2 D.

Example Determine if the line containing the points $(1,2)$ and $(-3,6)$ is parallel to the line containing the points $(5,-6)$ and $(-2,1)$.

Solution Using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$,

a. Let $(x_1,y_1) = (1,2)$ and $(x_2,y_2) = (-3,6)$.

$$m_1 = \frac{6-2}{-3-1} = \frac{4}{-4} = -1$$

b. Let $(x_1,y_1) = (5,-6)$ and $(x_2,y_2) = (-2,1)$.

$$m_2 = \frac{1-(-6)}{-2-5} = \frac{7}{-7} = -1$$

Since the slope of each line is the same, -1, the lines are parallel.

16. L_1 contains $(4,2)$ and $(1,-1)$
L_2 contains $(1,1)$ and $(0,0)$

17. L_1 contains $(5,-2)$ and $(4,1)$
L_2 contains $(0,6)$ and $(2,0)$

18. L_1 contains $(5,1)$ and $(-4,2)$
L_2 contains $(4,-3)$ and $(2,1)$

19. L_1 contains $(-6,3)$ and $(-2,-5)$
L_2 contains $(-4,1)$ and $(7,-6)$

Determine whether lines L_1 and L_2 are perpendicular. See example 7–2 E–1.

Example Determine if the lines containing the points $(1,-2)$ and $(3,5)$ and containing points $(-1,-1)$ and $(6,-3)$ are perpendicular.

Solution Using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$,

a. Let $(x_1,y_1) = (1,-2)$ and $(x_2,y_2) = (3,5)$.

$$m_1 = \frac{5-(-2)}{3-1} = \frac{7}{2}$$

b. Let $(x_1,y_1) = (-1,-1)$ and $(x_2,y_2) = (6,-3)$.

$$m_2 = \frac{-3-(-1)}{6-(-1)} = \frac{-2}{7} = -\frac{2}{7}$$

Since $\left(\dfrac{7}{2}\right)\left(-\dfrac{2}{7}\right) = -1$, the slopes are negative reciprocals and the lines are perpendicular.

20. L_1 contains $(2,-3)$ and $(4,3)$
L_2 contains $(1,0)$ and $(-2,1)$

21. L_1 contains $(5,-2)$ and $(-3,-3)$
L_2 contains $(4,6)$ and $(5,-2)$

22. L_1 contains $(1,1)$ and $(4,1)$
L_2 contains $(-2,2)$ and $(3,-3)$

23. L_1 contains $(-6,-6)$ and $(-1,-1)$
L_2 contains $(4,-4)$ and $(-4,4)$

Determine if the graphs of the given pairs of equations are parallel, perpendicular, or neither. See example 7–2 E–2.

24. $x + y = 1$ and $3x + 3y = -6$

25. $2x + y = -4$ and $6x + 3y = 12$

26. $y - x = -5$ and $4x + 4y = 8$

27. $4y - 3x = 12$ and $4x + 3y = 24$

28. $2y - 3x = 1$ and $3y + 2x = 5$

29. $x + 4y = 5$ and $2x - 5y = 2$

30. $3y - 4x = 1$ and $8y + 3x = 6$

31. $2y - x = 1$ and $6x + 3y = 0$

32. $x + 5y = 0$ and $3x + 15y = 2$

33. $3x - 7y = 0$ and $3y + 5x = 0$

Determine if the lines through the given sets of points are parallel, perpendicular, or neither.

34. (1,3) and (2,4); (7,2) and (8,3)

35. (−2,0) and (1,4); (5,2) and (9,5)

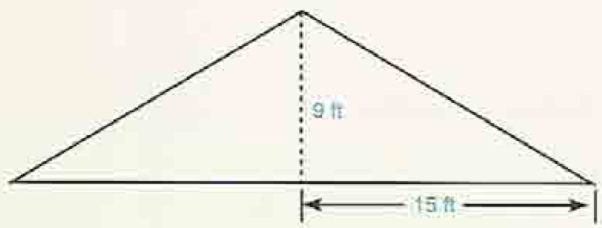
36. (1,5) and (−2,−3); (0,1) and (3,2)

37. (5,6) and (1,3); (−1,6) and (2,1)

38. (7,2) and (−3,4); (−1,−2) and (5,2)

Solve the following word problems.

39. The *pitch* of a roof is the slope of a roof. If the roof of a house rises vertically a distance of 9 feet through a horizontal distance of 15 feet, what is the pitch of the roof?



40. The roof of a school building rises 10 feet through a horizontal run of 35 feet. What is the pitch of the roof? (Refer to exercise 39.)

41. The guy wire of a telephone pole is attached to the pole 5 meters above the ground and attached to the ground 7.5 meters from the base of the pole. What is the slope of the guy wire?

42. A ladder leaning against a house touches the building at a point 14 feet above the ground. If the foot of the ladder is 9 feet from the base of the house, what is the slope of the ladder?

43. If a company's profits (P) are related to the number of items produced (x) by a linear equation, what is the slope of the graph of the equation if the profits rise by \$25,000 for every 175 items produced?

44. A company's profits (P) are related to increases in the workers' average pay (x) by a linear equation. If the company's profits drop by \$25,000 per year for every increase of \$550 per year in the workers' average pay, what is the slope of the graph of the equation?

45. The increase in a jogger's heartbeat in beats per minute is related to her increase in speed in feet per second by a linear equation. What is the slope of the graph of the equation if an increase in speed of 2 feet per second causes an increase of 10 heartbeats per minute and an increase in speed of 4 feet per second causes an increase of 25 heartbeats per minute? [*Hint:* Use ordered pairs (2,10) and (4,25).]

46. The bacteria count in a culture is related to the hours it exists by a linear equation. What is the slope of the graph if after $1\frac{1}{2}$ hours the bacteria count is 1,000,000 and after 2 hours the bacteria count is 4,500,000?

47. The decay of a substance is related by a linear equation to the time in years that the substance is left in the open air. If after 10 years there are 75 grams remaining and after 25 years there are 40 grams remaining, what is the slope of the equation?

48. The vertices of a triangle in the plane are at the points (−2,4), (5,2), and (0,−4). Find the perimeter (distance around) of the triangle.

49. Show that the points (4,2), (3,0), (−1,0), and (0,2) are the vertices of a parallelogram. Find the perimeter of the parallelogram. (*Hint:* A parallelogram is a four-sided figure whose opposite sides are parallel.)

50. Show that the points (−2,1), (5,3), (3,4), and (0,0) are the vertices of a parallelogram. Find the perimeter of the parallelogram. (Refer to exercise 49.)

51. Show that the points (4,2), (−2,−3), and (4,−3) are the vertices of a right triangle. (*Hint:* Show that two sides are perpendicular.)

52. Show that the points (2,−3), (5,1), and (−2,0) are the vertices of a right triangle by (a) using the Pythagorean Theorem and (b) showing two sides are perpendicular.

53. A trapezoid is a four-sided figure with one pair of opposite sides that are parallel. Show that the points (−3,2), (−1,−4), (5,2), and (9,−4) are the vertices of a trapezoid.

54. Three points that lie on the same straight line are said to be **collinear.** Three points are collinear if the sum of the distances between two pairs of points is equal to the distance between the third pair of points and if the slopes between all pairs of points are the same. Show that the points (6,7), (5,2), and (3,−8) are collinear using (a) slopes and (b) the distance formula.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55. Show that the points $(1,2)$, $(-3,-4)$, and $(-5,-7)$ are collinear using (a) slopes and (b) the distance formula. (Refer to exercise 54.)

$\boxed{56.}$ Find the abscissa of the points whose ordinate is -6 if the points are at a distance of $5\sqrt{5}$ from the point $(4,5)$.

57. Find the ordinate of the points whose abscissa is 4 if the points are at a distance of $2\sqrt{13}$ from the point $(-2,-7)$.

$\boxed{58.}$ Given one endpoint of a line segment is $(-2,3)$ and the midpoint of the line segment is $(2,-3)$, what are the coordinates of the other endpoint?

59. Given the midpoint of a line segment is the point $(1,5)$, find the other endpoint of the line segment if one endpoint is $(4,-2)$.

Review exercises

Solve the following equations for y. See section 2–2.

1. $3x + 2y = 4$

2. $4y - 3x = 8$

Solve the following inequalities for y. See section 2–2 and 2–5.

3. $4y + x < 8$

4. $x - 2y \geq 4$

Find the solution set of the following linear equations. See section 2–1.

5. $\frac{1}{2}x - 5 = \frac{2}{3}x + 1$

6. $3x = \frac{1}{2}(x - 2)$

7–3 ■ Finding the equation of a line

In section 7–1, we discussed the graph of a linear equation in two variables of the form $ax + by = c$ ($a \neq 0$ or $b \neq 0$). The graph of the equation is a straight line. In this section, we will determine the **equation of the graph** of a straight line. That is, we want an equation that is satisfied only by the coordinates of the points on the line. Thus the equation must be such that, for any arbitrary point P,

1. if P is on the graph, then its coordinates satisfy the equation, and
2. if P is not on the graph, then its coordinates do not satisfy the equation.

Consider a line in the plane having slope m that passes through a given point $P_1(x_1,y_1)$. Let $P(x,y)$ be any other arbitrary point on the line. See figure 7–13.

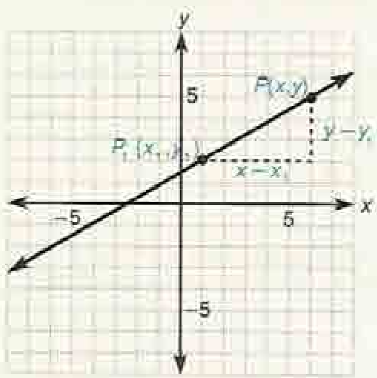


Figure 7–13

By the definition of the slope of a line, provided that $x \neq x_1$, we have

$$m = \frac{y - y_1}{x - x_1}$$

When we multiply each member by $x - x_1$, we obtain

$$y - y_1 = m(x - x_1)$$

We call this the **point-slope** form of the equation of a line.

Point-slope form

The point-slope form of the equation of a nonvertical line having slope m and passing through the known point (x_1,y_1) is given by

$$y - y_1 = m(x - x_1)$$

Example 7–3 A

1. Find the equation of the line having a slope of $-\frac{1}{2}$ and passing through the point $(-1,4)$.

   Use the point-slope form $y - y_1 = m(x - x_1)$.

   $y - (4) = \left(-\frac{1}{2}\right)[x - (-1)]$ Replace m with $-\frac{1}{2}$, y_1 with 4, and x_1 with -1

   $y - 4 = -\frac{1}{2}(x + 1)$

   $2y - 8 = -1(x + 1)$ Multiply each member by 2

   $2y - 8 = -x - 1$

   $x + 2y = 7$ Add $x + 8$ to each member

   We say that the equation $x + 2y = 7$ is written in **standard form.**

Standard form

The standard form of the equation of a line is written as

$$ax + by = c \ (a > 0)$$

where a, b, and c are real numbers and not both a and b equal to 0.

2. Find the equation of the line passing through the points $(2,4)$ and $(-5,-2)$. Write the equation in standard form.

   We must first find the slope m of the line. Using $m = \frac{y_2 - y_1}{x_2 - x_1}$, let $(x_1,y_1) = (2,4)$ and $(x_2,y_2) = (-5,-2)$.

   $$m = \frac{-2 - 4}{-5 - 2} = \frac{-6}{-7} = \frac{6}{7}$$

   Using the point-slope form $y - y_1 = m(x - x_1)$, the slope, and *one* of the given points on the line, $(2,4)$, we obtain

$$\begin{aligned}
y - (4) &= \left(\frac{6}{7}\right)[x - (2)] && \text{Replace } m \text{ with } \tfrac{6}{7},\ y_1 \text{ with 4, and } x_1 \text{ with 2} \\
7y - 28 &= 6(x - 2) && \text{Multiply each member by 7} \\
7y - 28 &= 6x - 12 \\
-6x + 7y &= 16 && \text{Add } -6x + 28 \text{ to each member} \\
6x - 7y &= -16 && \text{Multiply each member by } -1
\end{aligned}$$

The equation in standard form is $6x - 7y = -16$.

▶ ***Quick check*** Find the equation, in standard form, of the line passing through the points $(0,-7)$ and $(6,4)$. ■

Consider again the point-slope form of the equation of a nonvertical line. Given

$$y - y_1 = m(x - x_1)$$

suppose we let the known point be $(0,b)$, the y-intercept. Substituting 0 for x_1 and b for y_1, we have

$$\begin{aligned}
y - b &= m(x - 0) \\
y - b &= mx \\
y &= mx + b
\end{aligned}$$

We call this the **slope-intercept** form of the equation of a line.

Slope-intercept form

The slope-intercept form of the equation of a nonvertical line is written as

$$y = mx + b$$

where m is the slope and the point $(0,b)$ is the y-intercept.

■ Example 7–3 B

1. Find the slope and y-intercept of the line whose equation is $2y + 3x = 4$.

   To find the slope and y-intercept, we will write the equation in slope-intercept form, which is accomplished by solving the equation for y. Thus

$$\begin{aligned}
2y + 3x &= 4 \\
2y &= -3x + 4 && \text{Add } -3x \text{ to each member} \\
y &= -\frac{3}{2}x + 2 && \text{Divide each term by 2}
\end{aligned}$$

   Then the slope is $-\frac{3}{2}$ (the coefficient of x) and the y-intercept is the point $(0,2)$.

2. Find the equation of the line whose slope is $-\frac{5}{3}$ and whose y-intercept is $(0,3)$. Write the equation in standard form.

Using the slope-intercept form of the equation, $y = mx + b$, where $m = -\frac{5}{3}$ and $b = 3$,

$$\begin{aligned} y &= mx + b \\ y &= -\frac{5}{3}x + 3 && \text{Replace } m \text{ with } -\tfrac{5}{3} \text{ and } b \text{ with } 3 \\ 3y &= -5x + 9 && \text{Multiply each member by 3} \\ 5x + 3y &= 9 && \text{Add } 5x \text{ to each member} \end{aligned}$$

▶ ***Quick check*** Find the slope and y-intercept of the line whose equation is $8x + 5y = 10$.

We saw in the previous section that the slopes of two nonvertical *parallel* lines are the same and the slopes of two nonvertical *perpendicular* lines are negative reciprocals. We use these facts in the next two examples.

3. Find the equation of the line through the point $(3,1)$ that is parallel to the line whose equation is $3x - 2y = 4$.

For the lines to be parallel, the line we want must have the same slope as the line $3x - 2y = 4$. We solve this equation for y to obtain the slope-intercept form.

$$\begin{aligned} 3x - 2y &= 4 \\ -2y &= -3x + 4 && \text{Add } -3x \text{ to each member} \\ y &= \frac{3}{2}x - 2 && \text{Divide each member by } -2 \end{aligned}$$

Slope $m = \frac{3}{2}$

Using the point-slope form,

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (1) &= \left(\frac{3}{2}\right)[x - (3)] && \text{Replace } m \text{ with } \tfrac{3}{2}, y_1 \text{ with 1, and } x_1 \text{ with 3} \\ 2y - 2 &= 3(x - 3) && \text{Multiply each member by 2} \\ 2y - 2 &= 3x - 9 \\ 2y - 3x &= -7 && \text{Multiply each member by } -1 \\ 3x - 2y &= 7 && \text{Standard form} \end{aligned}$$

The equation of the line is $3x - 2y = 7$.

4. Find the equation of the line that is perpendicular to the line $4x - 2y = 5$ and passes through the point $(-3,4)$.

We write the equation $4x - 2y = 5$ in slope-intercept form to find its slope.

$$\begin{aligned} 4x - 2y &= 5 \\ -2y &= -4x + 5 && \text{Add } -4x \text{ to each member} \\ y &= 2x - \frac{5}{2} && \text{Divide each member by } -2 \end{aligned}$$

Slope $m = 2$

The slope of the line we desire is the negative reciprocal of 2, which is $-\frac{1}{2}$.

We want the equation of the line with slope $m = -\dfrac{1}{2}$ and passing through $(-3,4)$. Use the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - (4) = \left(-\frac{1}{2}\right)[x - (-3)] \qquad \text{Replace } m \text{ with } -\frac{1}{2},\ y_1 \text{ with } 4, \text{ and } x_1 \text{ with } -3$$

$$y - 4 = -\frac{1}{2}(x + 3)$$

$$2y - 8 = -1(x + 3) \qquad \text{Multiply each member by 2}$$

$$2y - 8 = -x - 3$$

$$x + 2y = 5$$

The equation of the line is $x + 2y = 5$.

5. Determine if the graphs of the equations $2x - 4y = 5$ and $4x + 2y = -5$ are perpendicular, using the slope-intercept form.

We must write each equation in slope-intercept form and compare the slopes. Solve for y.

a. $2x - 4y = 5$

$$-4y = -2x + 5 \qquad \text{Add } -2x \text{ to each member}$$

$$y = \frac{-2x + 5}{-4} \qquad \text{Divide each term by } -4$$

$$y = \frac{1}{2}x - \frac{5}{4} \qquad \text{Write in slope-intercept form}$$

$$m_1 = \frac{1}{2}$$

b. $4x + 2y = -5$

$$2y = -4x - 5 \qquad \text{Add } -4x \text{ to each member}$$

$$y = \frac{-4x - 5}{2} \qquad \text{Divide each term by 2}$$

$$y = -2x - \frac{5}{2} \qquad \text{Write in slope-intercept form}$$

$$m_2 = -2$$

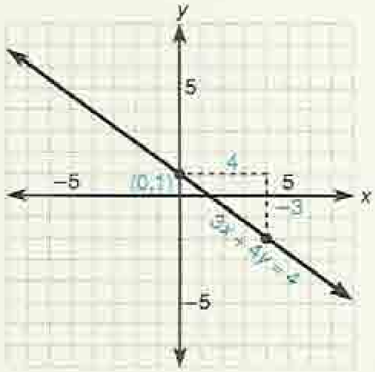
Since $\left(\dfrac{1}{2}\right)(-2) = -1$ (the slopes are negative reciprocals), the lines are perpendicular.

▶ ***Quick check*** Find the equation of the line through the point $(6,-7)$ that is parallel to the line whose equation is $4x - 5y = 3$. ■

The slope-intercept form of the equation of a line can be used to graph a linear equation in two variables.

■ Example 7–3 C

Graph the following equations using the slope and the y-intercept.



1. $3x + 4y = 4$

Write the equation in slope-intercept form.

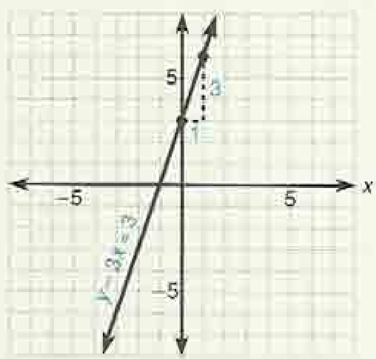
$$3x + 4y = 4$$
$$4y = -3x + 4 \quad \text{Add } -3x \text{ to each member}$$
$$y = -\frac{3}{4}x + 1 \quad \text{Divide each term by 4}$$

The slope is $m = -\frac{3}{4}$ and the y-intercept is $(0,1)$.

Recall that the definition of the slope of a line is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = -\frac{3}{4} = \frac{-3}{4}$$

a. Plot the y-intercept $(0,1)$.
b. From this point, move 4 units to the *right* (horizontal change) and then 3 units *down* (vertical change that is negative) to obtain another point on the graph.
c. Draw the line through the two points.



2. $y - 3x = 3$

Write the equation in slope-intercept form.

$$y = 3x + 3$$
$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = 3 = \frac{3}{1}$$

a. Plot the y-intercept $(0,3)$.
b. From this point, move 1 unit to the *right* (horizontal change) and then 3 units *up* (vertical change that is positive) to obtain a second point.
c. Draw the line through the two points.

▶ ***Quick check*** Graph $5y + 8x = 10$ using the slope and y-intercept. ■

We now summarize the different forms of a linear equation.

$ax + by = c (a > 0)$	Standard form
$y - y_1 = m(x - x_1)$	Point-slope form
$y = mx + b$	Slope-intercept form
$x = k$	Equation of a vertical line
$y = k$	Equation of a horizontal line

Mastery points

Can you

- Find the equation of a line using the point-slope form $y - y_1 = m(x - x_1)$?
- Write the equation of a line in standard form $ax + by = c$, $a > 0$?
- Write the equation of a line in slope-intercept form $y = mx + b$?
- Find the slope, m, and the y-intercept, b, of a line given its equation?
- Sketch the graph of a linear equation in two variables using the slope and y-intercept?

Exercise 7–3

Find the equation of the line that satisfies the given conditions. Write the equation in standard form. See example 7–3 A.

Example Through points $(0,-7)$ and $(6,4)$

Solution We first find the slope using the two points.

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{(4) - (-7)}{(6) - (0)} && \text{Replace } y_2 \text{ with } 4,\ y_1 \text{ with } -7,\ x_2 \text{ with } 6, \text{ and } x_1 \text{ with } 0 \\
&= \frac{4 + 7}{6} && \text{Definition of subtraction} \\
&= \frac{11}{6}
\end{aligned}
$$

Using the point-slope form,

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - (4) &= \left(\frac{11}{6}\right)[x - (6)] && \text{Replace } m \text{ with } \tfrac{11}{6},\ y_1 \text{ with } 4, \text{ and } x_1 \text{ with } 6 \text{ [using point } (6,4)] \\
6y - 24 &= 11(x - 6) && \text{Multiply each member by } 6 \\
6y - 24 &= 11x - 66 && \text{Distribute in the right member} \\
-11x + 6y &= -42 && \text{Add } -11x \text{ and } 24 \text{ to each member} \\
11x - 6y &= 42 && \text{Multiply each term by } -1
\end{aligned}
$$

1. Slope $m = \frac{1}{2}$ and passing through $(3,4)$
2. Slope $m = -\frac{5}{4}$ and passing through $(-1,-6)$
3. Slope $m = 5$ and passing through $(0,7)$
4. Slope $m = -\frac{3}{2}$ and having x-intercept -5
5. Slope $m = -\frac{5}{6}$ and passing through $(0,0)$
6. Slope $m = -6$ and having y-intercept 2
7. Slope $m = 1$ and having y-intercept -3
8. Horizontal line through $(5,-3)$
9. Horizontal line through $(1,4)$
10. Horizontal line through $(0,0)$
11. Slope is undefined and passing through $(-5,6)$
12. Slope is undefined and passing through $(1,2)$

13. Vertical line passing through $(5,-4)$

14. Vertical line passing through $(-6,2)$

15. Vertical line passing through $(0,0)$

16. Slope $m = 0$ and passing through $(3,5)$

17. Slope $m = 0$ and passing through $(0,-7)$

Find the equation of the line through the given points. Write the equation in standard form.

18. $(1,2)$ and $(5,4)$
19. $(3,6)$ and $(1,1)$
20. $(-3,4)$ and $(5,-1)$
21. $(2,6)$ and $(-1,-4)$
22. $(5,0)$ and $(-2,-3)$
23. $(4,0)$ and $(5,-2)$
24. $(3,-6)$ and $(2,-6)$
25. $(5,4)$ and $(-2,4)$
26. $(-2,-2)$ and $(4,4)$
27. $(-4,4)$ and $(3,-3)$
28. $(-5,5)$ and $(1,-1)$
29. $(4,-3)$ and $(4,-7)$
30. $(-2,6)$ and $(-2,-5)$

Write each equation in slope-intercept form $y = mx + b$ and identify the slope m and the y-intercept b. Sketch the graph using the slope and y-intercept. See examples 7–3 B and C.

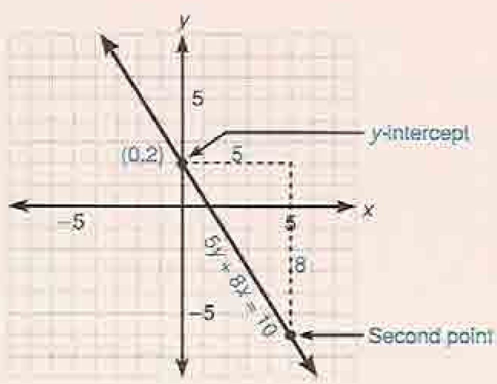
Example $5y + 8x = 10$

Solution We first solve the equation for y.

$$5y + 8x = 10 \quad \text{Original equation}$$
$$5y = -8x + 10 \quad \text{Add } -8x \text{ to each member}$$
$$y = -\frac{8}{5}x + 2 \quad \text{Divide each term by 5}$$

The slope is $m = -\frac{8}{5}$ and the y-intercept is $(0,2)$.

1. Plot the y-intercept $(0,2)$.
2. From this point, since $m = \frac{-8}{5}$ move 5 units to the right (horizontal change) and 8 units *down* (vertical change is *negative*) to obtain a second point.
3. Draw the straight line through these two points.



31. $2x + y = 5$
32. $3x + y = -3$
33. $4x - y = 6$
34. $3x - 7y = 0$
35. $5y - 2x = 0$
36. $2y + 9x = 0$
37. $8x + 3y = 0$
38. $x - y = 0$
39. $3y + 5 = 0$
40. $2y - 7 = 0$

Find the equation of the line satisfying the given conditions. Write each equation in standard form. See example 7–3 B–3 and 4.

Example Through the point $(6,-7)$ and parallel to the line $4x - 5y = 3$

Solution Find the slope of the given line. Solve for y.

$$4x - 5y = 3 \quad \text{Original equation}$$
$$-5y = -4x + 3 \quad \text{Subtract } 4x \text{ from each member}$$
$$y = \frac{4}{5}x - \frac{3}{5} \quad \text{Divide each term by } -5$$

The slope $m = \frac{4}{5}$. Since we want a line parallel to the given line, the slope will be the same.

Using the point-slope form,

$$y - y_1 = m(x - x_1)$$
$$y - (-7) = \left(\frac{4}{5}\right)[x - (6)] \quad \text{Replace } m \text{ with } \frac{4}{5},\ y_1 \text{ with } -7, \text{ and } x_1 \text{ with } 6$$
$$y + 7 = \frac{4}{5}(x - 6) \quad \text{Double negative property in the left member}$$
$$5y + 35 = 4(x - 6) \quad \text{Multiply each member by 5}$$
$$5y + 35 = 4x - 24 \quad \text{Distribute in right member}$$
$$-4x + 5y = -59 \quad \text{Subtract } 4x \text{ and } 35 \text{ from each member}$$
$$4x - 5y = 59 \quad \text{Multiply each term by } -1$$

41. Parallel to $3x + y = 6$ and passing through $(1,2)$

42. Perpendicular to $2x + 5y = 3$ and passing through $(1,7)$

43. Passing through $(-3,-1)$ and parallel to $3x - 2y = 9$

44. Passing through $(-7,4)$ and perpendicular to $7y - 2x = 0$

45. Passing through $(0,0)$ and perpendicular to $9x - 2y = 1$

46. Passing through $(5,-3)$ and parallel to $y - 2 = 0$

47. Passing through $(4,0)$ and perpendicular to $y = 5$

48. Find the equation of the perpendicular bisector of the line segment whose endpoints are $(-2,3)$ and $(4,7)$. (*Hint:* We want the line perpendicular at the midpoint.)

49. Find the equation of the perpendicular bisector of the line segment whose endpoints are $(3,-4)$ and $(3,6)$.

Using the slope-intercept form of the equation of a line, determine if the given pairs of equations represent parallel lines, perpendicular lines, or neither. See example 7–3 B–5.

50. $x + 2y = 5$ and $6x - 3y = 4$

51. $3y + 2x = 5$ and $2y + 3x = -1$

52. $2y - 5x = -3$ and $y + 3x = 4$

53. $4y - 5x = -1$ and $8x + 10y = 4$

54. $3x + 8y = 2$ and $6x - 16y = 5$

55. $8x - 5y = 1$ and $16x - 10y = 7$

Solve the following word problems.

56. Given the point-slope form of the equation of a line,

$y - y_1 = m(x - x_1)$

if the points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ lie on the graph, then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

and if we substitute, we obtain the **two-point** form of the equation of a line given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Using this form, find the equation in standard form for the line passing through the given points (a) (3,2) and (4,1); (b) (5,−1) and (−3,0).

57. Given the two-point form of the equation of a line found in exercise 56, use the points $(a,0)$ and $(0,b)$ to show that

$$\frac{x}{a} + \frac{y}{b} = 1$$

is an equation of the line with x-intercept $(a,0)$ and y-intercept $(0,b)$. We call this the **intercept** form of the equation of a line.

58. Write each equation in **intercept** form and determine the x- and y-intercepts. (Refer to exercise 57.)

a. $4x + 3y = 12$ b. $2x + 5y = 10$
c. $3x - y = 3$ d. $3x - 5y = 6$
e. $5y - 2x = 8$ f. $6x + 5y = 12$

59. Find the slope and y-intercept of the equation $ax + by = c$ in terms of a, b, and c.

60. What is the slope of a line that is parallel to the line with the equation $ax + by = c$?

61. What is the slope of a line that is perpendicular to the line with the equation $ax + by = c$?

There are a number of real-life situations that can be described using linear equations in two variables. If two pairs of values are known, we can use the *two-point* form demonstrated in exercise 56. Express each equation in standard form.

62. A company produces 300 boxes of cereal for \$150 and 600 boxes of the same cereal for \$250. Let x be the number of boxes of cereal and y be the total cost.

63. John Doe makes \$150 profit on 4 waterfront paintings that he does and \$400 profit on 7 paintings. Let x be the number of paintings and y be the profit on the paintings.

64. A company found its total sales were \$35,000 in the third year of operation and \$105,000 in the fifth year of operation. Let x be the year of operation and y the total sales that year.

65. Use the resulting equation of exercise 64 to predict the total sales in the sixth year.

66. Use the resulting equation of exercise 62 to determine the approximate cost of producing 1,000 boxes.

Review exercises

Solve the following inequalities. See sections 2–2 and 2–5.

1. $3x - 2 \leq x - 1$

2. $4y - 3x > 6$ (for y)

Graph the following equations. See section 7–1.

3. $x - 3y = 6$

4. $2y + x = -4$

5. Find the solution set of the quadratic equation $2x^2 - 3x = 4$. See section 6–3.

6. Find the solution set of the radical equation $\sqrt{x + 3} = x - 3$. See section 6–5.

7–4 ■ Graphs of linear inequalities

Linear inequalities in two variables

In chapter 2, we discussed the solution set of linear inequalities in one variable such as

$$x + 5 < 3, \quad 2x - 3 \le 1, \quad 4x - 3 > 0, \quad \text{and} \quad -4 \le 2x + 1 < 3$$

In this chapter, we have discussed the graph of the solution set of linear equations in two variables such as

$$2x - 3y = 5, \quad x - 2y = 6, \quad y = -4, \quad \text{and} \quad x = 5$$

Now we extend these ideas to consider the solution set and the graph of **linear inequalities in two variables** such as

$$y \le x, \quad 3x - 2y > 4, \quad \text{and} \quad y < 2x + 3$$

Linear inequalities in two variables

Any inequality of the form

$$\begin{array}{lcl} ax + by < c, & & ax + by > c, \\ ax + by \le c, & \text{or} & ax + by \ge c, \end{array}$$

where a, b, and c are real numbers, a and b not both zero, is called a linear inequality in two variables.

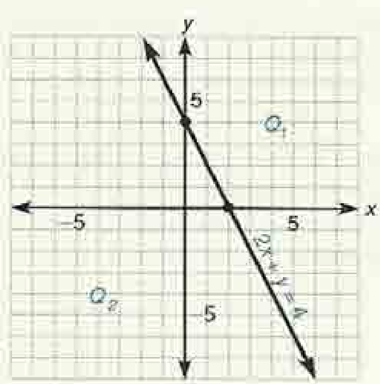
The graph of any linear inequality in two variables will be a *half-plane*, as illustrated in the following examples.

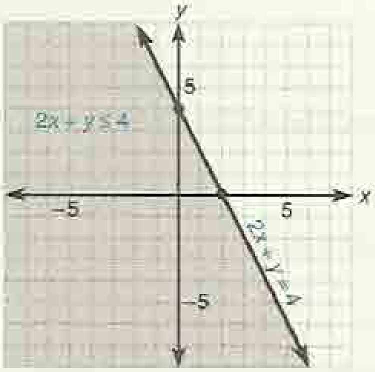
■ **Example 7–4 A**

Graph the following linear inequalities.

1. $2x + y \le 4$

To graph the inequality $2x + y \le 4$, we first graph the straight line $2x + y = 4$.



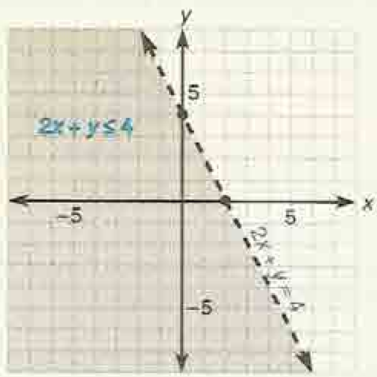


The graph of the inequality includes the points of this line together with all points of the plane either *above* or *below* this line. To decide which half-plane is the solution set, choose *any point not on the line* $2x + y = 4$ and substitute these values into the inequality $2x + y \leq 4$. *The origin, (0,0), is most often a good choice.* Replacing both x and y with 0 in the inequality $2x + y \leq 4$,

$$2(0) + (0) \leq 4 \quad \text{Replace } x \text{ with 0 and } y \text{ with 0}$$
$$0 \leq 4 \quad \text{(True)}$$

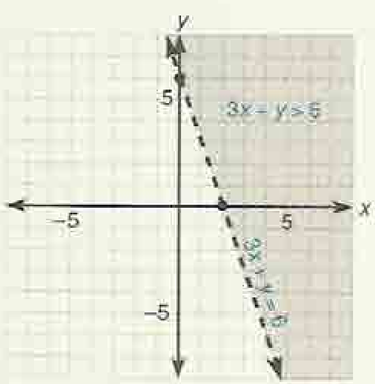
Since the result is true, (0,0) does satisfy the inequality and the solution set includes all points on the side of the line where (0,0) lies. We then shade that half-plane.

Note If our inequality had been the *strict* inequality $2x + y < 4$, instead of the *weak* inequality $2x + y \leq 4$, the line $2x + y = 4$ would be *dashed* since the points of the line are not in the solution set anymore.



2. $3x + y > 6$

Graph the equation $3x + y = 6$ and make the line dashed since the order symbol, $>$, does not include equality. Since (0,0) does not lie on the line, choose this as a test point and substitute 0 for x and 0 for y in the inequality.



$$3x + y > 6 \quad \text{Original inequality}$$
$$3(0) + (0) > 6 \quad \text{Replace } x \text{ with 0 and } y \text{ with 0}$$
$$0 + 0 > 6$$
$$0 > 6 \quad \text{(False)}$$

Since (0,0) does not satisfy the inequality, shade the half-plane that *does not* contain the origin.

▶ ***Quick check*** Graph $4x + y > 8$. ■

To graph a linear inequality in two variables

1. Replace the inequality symbol by the equality symbol.
2. Graph this line making it (1) a solid line if the inequality symbol is $\leq$ or $\geq$ (which includes the line in the solution set) or (2) a dashed line if the inequality symbol is $<$ or $>$ (which does *not* include the line in the solution set).
3. Choose some test point that is *not* on the line [if possible, the origin (0,0) since the arithmetic is easiest for this point].
4. Substitute the coordinates of the test point in the inequality.
5. If the test point's coordinates satisfy the inequality, shade the half-plane containing that point for the solution set; if the test point's coordinates *do not* satisfy the inequality, shade the other half-plane for the solution set.

■ Example 7–4 B

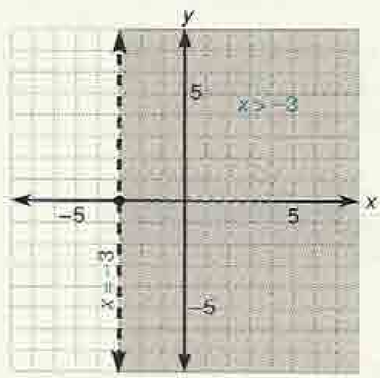
Graph the following inequalities.

1. $x > -3$

Graph the vertical line $x = -3$ with a dashed line since the order symbol is $>$ and does not include equality. Choose test point (0,0) and substitute 0 for x in the inequality.

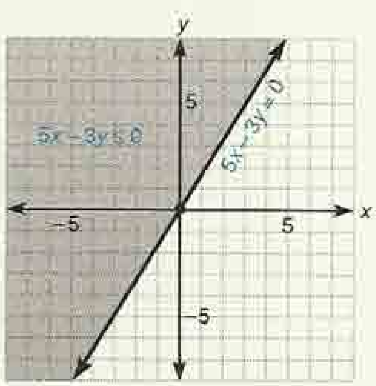
$x > -3$
$0 > -3$ Replace x with 0 (True)

Shade the half-plane containing the origin.



2. $5x - 3y \leq 0$

Graph the line $5x - 3y = 0$ and make it a solid line. The point (0,0) is on the line since the graph passes through the origin. Therefore we cannot use (0,0) as a test point and we arbitrarily choose some point that does not lie on the line. Suppose we choose (3,0). We substitute 3 for x and 0 for y in the inequality.



$$\begin{aligned} 5x - 3y &\leq 0 \\ 5(3) - 3(0) &\leq 0 \\ 15 - 0 &\leq 0 \\ 15 &\leq 0 \quad \text{(False)} \end{aligned}$$

Since the statement is false, shade the half-plane that *does not* contain (3,0).

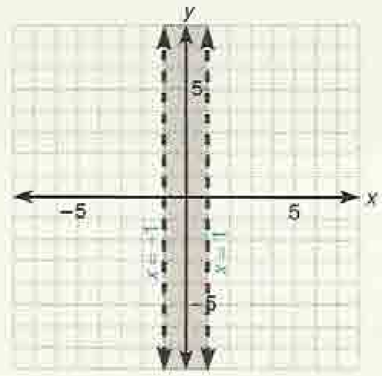
3. $|x| < 1$

In section 2–5, we learned that if

a. $|x| < 1$, then $-1 < x < 1$

b. $|x| = 1$, then $x = 1$ or $x = -1$.

Thus, $x = 1$ and $x = -1$ are boundary lines (dashed), and we want all points whose first coordinate (x-value) lies between 1 and -1. Determine this by checking points in all three regions $x < -1$, $-1 < x < 1$, and $x > 1$.



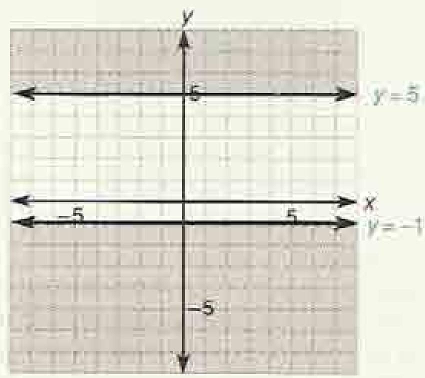
4. $|y - 2| \geq 3$

In section 2–5, we learned that if

a. $|y - 2| \geq 3$, then $y - 2 \geq 3$ or $y - 2 \leq -3$
$$y \geq 5 \qquad y \leq -1$$

b. $y - 2 = 3$, then $y - 2 = 3$ or $y - 2 = -3$
$$y = 5 \qquad y = -1$$

Thus, $y = 5$ and $y = -1$ are boundary lines (solid), and we want all points whose second coordinate (y-value) is greater than 5 or less than -1. *Check all three regions $y < -1$, $-1 < y < 5$, and $y > 5$.*



▶ ***Quick check*** Graph $|x + 3| \geq 1$.

Mastery points

Can you

- Determine when the boundary line is solid and when it is dashed?
- Graph the solution set of a linear inequality in two variables?
- Graph an absolute value inequality?

Exercise 7–4

Graph the solution set of each linear inequality. See examples 7–4 A and B.

Example $4x + y > 8$

Solution 1. Graph the line $4x + y = 8$ as a dashed line since we have $>$.

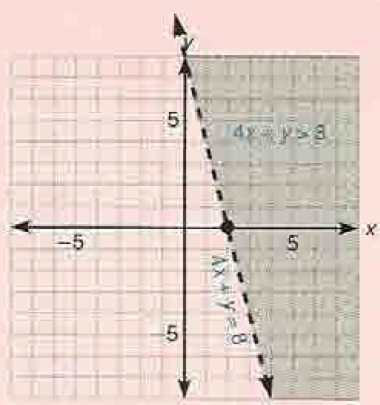
2. Choose test point (0,0) since the boundary line does not go through the origin.

$4(0) + 0 > 8$ Replace x with 0 and y with 0

$0 + 0 > 8$ Multiply in left member

$0 > 8$ (False)

3. Shade the half-plane that *does not* contain (0,0).



1. $y < 3$
2. $y \leq -4$
3. $x \geq 1$
4. $x > -5$
5. $y > 2$
6. $y \geq -1$
7. $x - 7 \geq -3$
8. $x + y > 5$
9. $x + y < -1$
10. $y < 3x - 1$
11. $y > 4 - 3x$
12. $x \leq 2y$
13. $3y > 4x$
14. $2y \geq -x$
15. $2x - 3y \leq 0$
16. $y - x > 0$
17. $5x + 2y \geq -10$
18. $3y - 5x > -15$

Graph the following absolute value inequalities in two variables. See example 7–4 B–3 and 4.

Example $|x + 3| \geq 1$

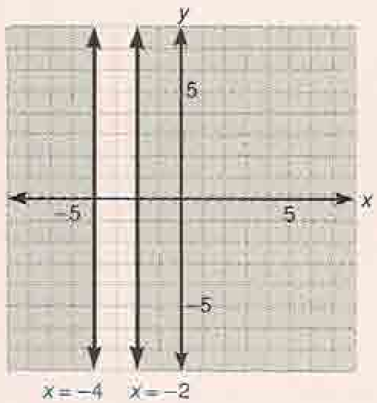
Solution By property in section 2–5, if $|x + 3| \geq 1$, then $x + 3 \geq 1$ or $x + 3 \leq -1$

$x \geq -2$ $\quad$ $x \leq -4$

If $|x + 3| = 1$, then $x + 3 = 1$ or $x + 3 = -1$

$x = -2$ and $x = -4$

The boundary lines are the solid lines $x = -2$ and $x = -4$. The solutions are all points whose first component (x-value) is less than -4 or greater than -2.



$\boxed{19.}$ $|x| < 2$ **20.** $|x| > 3$ **21.** $|x| \geq 1$ **22.** $|x| \leq 4$ **23.** $|y| < 1$

24. $|y| > 2$ **25.** $|y| \geq 3$ **26.** $|y| \leq 5$ **27.** $|x - 1| \leq 3$ **28.** $|x + 2| < 1$

29. $|x - 5| > 2$ $\boxed{30.}$ $|3 - x| \geq 4$ **31.** $|y - 4| \leq 3$ **32.** $|y + 6| < 2$ **33.** $|y + 7| > 1$

34. $|5 - y| \geq 3$ $\boxed{35.}$ $|2x - 3| \leq 3$ **36.** $|3y + 1| < 5$

Review exercises

Graph the following set of equations on the same coordinate axes. Determine the point of intersection. See section 7–1.

1. $2x - y = 3$
$x + y = 3$

2. $x - 3y = 1$
$x + 2y = 6$

3. Given $P(x) = 4x^2 - 2x + 1$, find $P(-1)$ and $P(2)$. See section 1–5.

4. Simplify the expression $2^2 - [3(5 - 1) + 4(6 + 2)]$. See section 1–4.

Perform the indicated operations. Reduce to lowest terms. See sections 4–2 and 4–3.

5. $\dfrac{3x - 1}{x + 3} - \dfrac{x - 1}{x^2 - 9}$

6. $\dfrac{4x^3}{9y^2} \div \dfrac{16x}{3y}$

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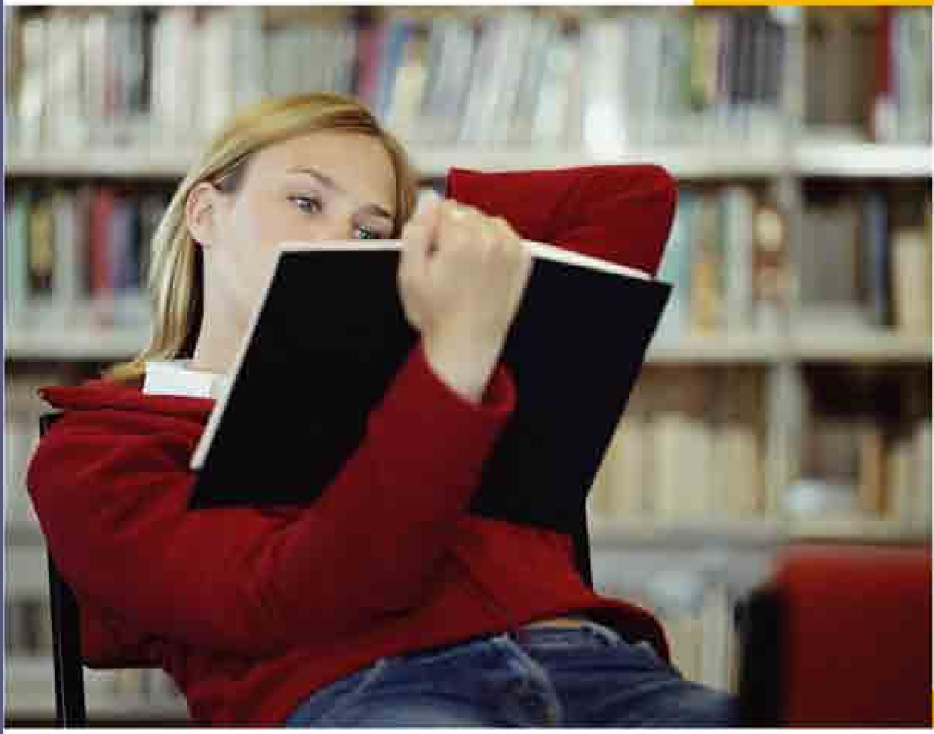

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Chapter 7 lead-in problem

A company that manufactures a heating unit can produce 20 units for \$13,900 while it would cost \$7,500 to manufacture 10 units. Assume the cost and number of units produced are related by the linear equation of a straight line. Let y be the total cost to manufacture x units. Find the linear equation of the straight line.

Solution

Using ordered pairs (x,y), we are given the known ordered pairs (20, 13,900) and (10, 7,500). We first find the slope of the line using the ordered pairs.

$$
\begin{aligned}
m &= \frac{y_1 - y_2}{x_1 - x_2} \\
&= \frac{13{,}900 - 7{,}500}{20 - 10} && \text{Replace } y_1 \text{ with } 13{,}900,\ y_2 \text{ with } 7{,}500,\ x_1 \text{ with } 20, \text{ and } x_2 \text{ with } 10 \\
&= \frac{6{,}400}{10} \\
&= 640
\end{aligned}
$$

Using the point-slope form of a line and one of the ordered pairs, (10, 7,500), we find the equation of the line.

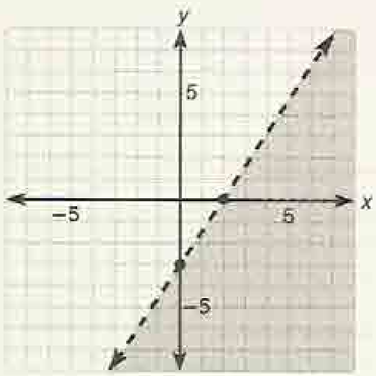
$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 7{,}500 &= 640(x - 10) && \text{Replace } m \text{ with } 640,\ y_1 \text{ with } 7{,}500, \text{ and } x_1 \text{ with } 10 \\
y - 7{,}500 &= 640x - 6{,}400 && \text{Distribute in the right member} \\
640x - y &= -1{,}100 && \text{Write in standard form}
\end{aligned}
$$

Chapter 7 summary

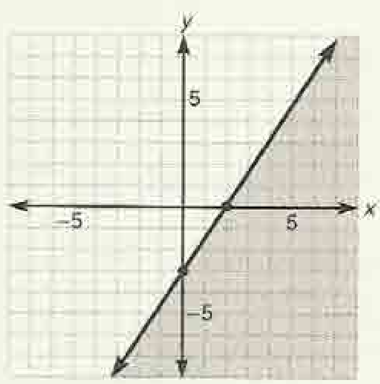
1. An **ordered pair** of real numbers is written as (x,y), where x and y are real numbers.
2. The **rectangular coordinate system** is formed by two real number lines, called **axes,** drawn in the plane—one horizontal (x-axis) and the other vertical (y-axis)—that intersect at the **origin** 0 of each line.
3. The axes divide the plane into four regions called **quadrants.**
4. Each point in the rectangular coordinate system is associated with only one ordered pair of real numbers, called the **coordinates of the point.** The point is called the **graph** of the ordered pair.
5. In the ordered pair (x,y), x is called the **first component** and y is called the **second component** of the ordered pair.
6. The **abscissa** of a point in the plane is the first component of the ordered pair and the **ordinate** of a point is the second component of the ordered pair.
7. The **graph of an equation** is the set of points in the plane associated with the solutions of the equation.
8. The graph of the linear equation in two variables $ax + by = c$ is a **straight line.**
9. The abscissa of the point at which a line intersects the x-axis is called the **x-intercept** of the line.
10. The **y-intercept** of a line is the **ordinate** of the point at which the line intersects the y-axis.
11. The distance between two points in the plane, $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, denoted by d, is given by
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
We call this the **distance formula.**
12. The **slope** m of the line containing $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by
$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{y_1 - y_2}{x_1 - x_2} \quad (x_1 \neq x_2)$$
13. Two nonvertical lines are **parallel** if and only if they have the same slopes.
14. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals.
15. The **point-slope** form of the equation of a nonvertical line having slope m and passing through (x_1,y_1) is given by $y - y_1 = m(x - x_1)$.
16. The **slope-intercept** form of the equation of a nonvertical line is written as $y = mx + b$, where m is the slope and b is the y-intercept.
17. The graph of a linear inequality in two variables, $ax + by > c$, $ax + by \geq c$, $ax + by < c$, or $ax + by \leq c$, is the **half-plane** on one side of the line $ax + by = c$. The graph includes the line if we have a weak inequality ($\leq$ or $\geq$), otherwise, it does not.

Chapter 7 error analysis

1. Finding the midpoint of a line segment
   Example: Endpoints are $(2,-3)$ and $(-1,5)$
   Midpoint $= \left(\frac{2-(-1)}{2}, \frac{-3-5}{2}\right) = \left(\frac{3}{2}, -4\right)$
   Correct answer: $\left(\frac{1}{2}, 1\right)$
   What error was made? (*see page 316*)
2. Finding the slope of a line
   Example: Containing points $(2,-5)$ and $(6,-3)$
   $m = \frac{-5-(-3)}{6-2} = \frac{-5+3}{4} = \frac{-2}{4} = -\frac{1}{2}$
   Correct answer: $m = \frac{1}{2}$
   What error was made? (*see page 318*)
3. The slope of a line
   Example: The slope of the line through $(5,-2)$ and $(5,3)$ is $m = 0$ since $x_1 = x_2 = 5$.
   Correct answer: m is undefined.
   What error was made? (*see page 320*)
4. Graphing linear inequalities
   Example: The graph of $2y - 3x \leq -6$ is



Correct answer:



What error was made? (*see page 338*)

5. Finding the distance between two points
   Example: Find the distance between the points $(-1,3)$ and $(2,5)$.
   $d = \sqrt{(-1+2)^2 + (3+5)^2} = \sqrt{1^2 + 8^2} = \sqrt{65}$
   Correct answer: $\sqrt{13}$
   What error was made? (*see page 315*)
6. Squaring a radical binomial
   Example: $(3\sqrt{2} - 5\sqrt{3})^2 = (3\sqrt{2})^2 - (5\sqrt{3})^2$
   $= 18 - 75 = -57$
   Correct answer: $93 - 30\sqrt{6}$
   What error was made? (*see page 242*)
7. Combining like terms
   Example: $a^2b - 2ab^2 - 6a^2b + ab^2$
   $= (1+6)a^2b - (2+1)ab^2 = 7ab^2 - 3ab^2$
   Correct answer: $-5a^2b - ab^2$
   What error was made? (*see page 41*)
8. Solving absolute value inequalities
   Example: $|2x - 3| > 7$
   $$-7 < 2x - 3 < 7$$
   $$-4 < 2x < 10$$
   $$\{x \mid -2 < x < 5\}$$
   Correct answer: $\{x \mid x < -2 \text{ or } x > 5\}$
   What error was made? (*see page 88*)
9. Perpendicular lines
   Example: The lines $3x - y = 7$ and $6x + 2y = 5$ are perpendicular.
   Correct answer: The lines are not perpendicular.
   What error was made? (*see page 322*)
10. Reducing a rational expression
    Example: $\frac{3x^2 - x^3}{x^3} = \frac{3x^2 - \cancel{x^3}^{\,1}}{\cancel{x^3}_{\,1}} = 3x^2 - 1$
    Correct answer: $\frac{3-x}{x}$
    What error was made? (*see page 157*)

Chapter 7 critical thinking

If there are n points in a plane where no three points lie on the same line, how many lines can be drawn through the n points?

Chapter 7 review

[7–1]

Plot the graph of each ordered pair of real numbers and state the quadrant in which the point lies, if it lies in a quadrant.

1. $(-3,5)$
2. $(-2,-7)$
3. $(0,-1)$
4. $(5,0)$
5. $\left(1,-\frac{3}{2}\right)$
6. $\left(\frac{1}{2},\frac{11}{2}\right)$
7. $(0,4)$
8. $\left(-\frac{5}{2},-4\right)$

Find the x- and y-intercepts of the graph for each equation (if they exist).

9. $x - 3y = 6$
10. $2x - y = 8$
11. $5x - 2y = 10$
12. $x = -6$
13. $y = 5$
14. $4y + 2x = 7$

Sketch the graph of each equation using the x- and y-intercepts (if they exist).

15. $y = 2x - 3$
16. $y = -5x + 1$
17. $y = 3x$
18. $x - 2y = 4$
19. $3y - 2x = 9$
20. $y = -6$
21. $x = 1$

[7–2]

Find the distance, midpoint, and slope between each pair of points.

22. $(3,2)$ and $(0,4)$
23. $(-2,1)$ and $(-2,4)$
24. $(1,5)$ and $(-3,5)$

Determine if the lines L_1 and L_2 containing the given points are parallel, perpendicular, or neither.

25. L_1 contains $(-3,2)$ and $(1,3)$
L_2 contains $(5,-1)$ and $(3,2)$
26. L_1 contains $(0,-1)$ and $(2,3)$
L_2 contains $(3,2)$ and $(1,3)$
27. L_1 contains $(4,0)$ and $(-3,-4)$
L_2 contains $(5,6)$ and $(-2,2)$
28. L_1 contains $(0,0)$ and $(9,-3)$
L_2 contains $(-1,-3)$ and $(8,2)$

Determine if the graphs of each pair of equations are parallel lines, perpendicular lines, or neither.

29. $2x + y = 1$ and $4x + 2y = -6$
30. $x - 3y = -2$ and $2x + 3y = 0$
31. $2x - 5y = 3$ and $5x + 2y = 1$
32. $3x - 2y = 1$ and $6x + 9y = 3$
33. A brace for a wall shelf is attached to the wall 9 inches below where the shelf meets the wall and to the bottom of the shelf $6\frac{1}{2}$ inches from the wall. What is the slope of the brace?
34. Show that the points $(-3,1)$, $(4,1)$, and $(-3,4)$ are the vertices of a right triangle.

[7–3]

Find the equation of the line satisfying the given conditions. Write the equation in standard form $ax + by = c$, $a > 0$.

35. Slope $m = \frac{2}{3}$ and passing through $(-1,5)$
36. Horizontal line through $(-2,4)$
37. Undefined slope and passing through $(1,-9)$
38. Passing through points $(2,-5)$ and $(3,3)$

Write each equation in slope-intercept form $y = mx + b$, identify the slope m and y-intercept b, and sketch the graph of the equation using the slope and y-intercept.

39. $3x - y = 4$ **40.** $2x + 3y = 9$ **41.** $3x - 2y = 0$ **42.** $2y + 3 = 0$

43. Find the equation of the line (in standard form) that passes through the point $(-7,2)$ and is parallel to the line $2y - x = 3$.

44. Find the equation of the line (in standard form) that is perpendicular to the line $4x + 5y = -2$ and passing through the point $(0,3)$.

[7–4]

Sketch the graph of each linear inequality in two variables.

45. $x \geq 0$ **46.** $y < -1$ **47.** $x + 3 < 0$ **48.** $y - 4 \geq 0$

49. $x + y \geq 4$ **50.** $3x - y < 6$ **51.** $x + 4y \geq 8$ **52.** $2x + 7y \leq 14$

Sketch the graph of each absolute value inequality.

53. $|x| \leq 5$ **54.** $|y| > 1$ **55.** $|x + 3| < 2$ **56.** $|x - 4| \geq 1$

57. $|y + 3| \leq 1$ **58.** $|2x - 3| \geq 1$

Chapter 7 cumulative test

Perform the indicated operations and simplify. Assume all variables are nonzero real numbers. Answer with positive exponents only.

[3–1] **1.** $(3a^2b^3)(-6ab^2)$

[3–3] **2.** $\dfrac{x^{-3}y^2}{x^2y^{-4}}$

[3–3] **3.** $(4a^{-3}b^2c^{-1})^{-2}$

[1–6] **4.** $5y^2 - \{5y - [4y^2 + 2y - 3]\}$

[3–2] **5.** $(x - 2y)^2 - (2x + y)^2$

[3–2] **6.** $6xy(2x^2 - 3y^2 + x^2y - x^3y^3)$

Completely factor the following expressions.

[3–7] **7.** $3a^2 - 12b^2$

[3–5] **8.** $8x^2 - 55x - 7$

[3–7] **9.** $8x^3 + 27y^3$

[3–5] **10.** $16x^2 - 40xy + 25y^2$

Find the solution set of the following equations and inequalities.

[2–1] **11.** $3(2a - 5) - 5a = 5(a + 6)$

[2–4] **12.** $|2x + 3| = 5$

[2–6] **13.** $|2y - 1| < 4$

[2–6] **14.** $|y + 3| \geq 1$

[6–1] **15.** $4x^2 - x = 0$

[6–1] **16.** $x^2 - 7x - 18 = 0$

[6–3] **17.** $4y^2 = 2y + 6$

[2–5] **18.** $-5 < 4 - 3x \leq 5$

[6–3] **19.** $2x^2 - 3x = 7$

Perform the indicated operations and simplify. Assume all denominators are nonzero.

[4–2] **20.** $\dfrac{y + 2}{y^2 - y - 20} \cdot \dfrac{y^2 - 16}{y^2 - 2y - 8}$

[4–3] **21.** $\dfrac{7y - 3}{2y^2 - 14y} - \dfrac{5y}{y^2 - 49}$

[4–4] **22.** Simplify the complex fraction $\dfrac{\dfrac{4}{y} - \dfrac{3}{x}}{\dfrac{4x - 3y}{xy}}$.

[4–7] **23.** Find the solution set of the equation $\dfrac{3}{8x - 1} = \dfrac{2}{2x + 3}$.

[4–6] **24.** Divide $3x^3 + 8x^2 - 7x - 12$ by $(x + 3)$. Use the results to (a) find $P(-3)$ when $P(x) = 3x^3 + 8x^2 - 7x - 12$ and (b) determine if $x + 3$ is a factor of $3x^3 + 8x^2 - 7x - 12$.

Perform the indicated operations, simplify, and rationalize all denominators.

[5–3] **25.** $\sqrt{18} + \sqrt{50} - \sqrt{8}$

[5–6] **26.** $(4 + \sqrt{5})^2$

[5–4] **27.** $\sqrt{\dfrac{9}{5}}$

[5–4] **28.** $\dfrac{3}{2 - \sqrt{3}}$

[5–7] **29.** $(3 + 2i)(3 - 2i)$

[5–7] **30.** $(4 - 3i) - (2 + 4i)$

[6–5] **31.** Solve the equation $\sqrt{x - 3} - 1 = \sqrt{x + 2}$. Name any extraneous solutions.

In problems 32–34, find the equation of the line (in standard form) satisfying the following conditions.

[7–3] **32.** Passing through points $(1, -3)$ and $(2, 4)$

[7–3] **33.** Passing through $(-1, 2)$ and parallel to $2x - 3y = 1$

[7–3] **34.** A vertical line passing through $(5, -3)$

[7–3] **35.** Find the slope and y-intercept of the line $3x - 5y = 10$. Sketch the graph using these facts.

[7–3] **36.** Are the lines $2x - y = 6$ and $4x + 8y = 1$ parallel, perpendicular, or neither?

[7–2] **37.** Find the distance between the points $(1, -3)$ and $(2, 5)$. Find the coordinates of the midpoint of the line segment.

Chapter 6 cumulative test

1. -12 **2.** 2 **3.** 56 **4.** $7xy^2 - 6xy + 6x^2y$
5. $9x^2 + 12xy + 4y^2$ **6.** $16y^2 - 1$ **7.** $9x^3 - 16x^2 - 8$
8. $-3{,}375x^6y^9$ **9.** a^{20} **10.** $\frac{-2b^5}{a^5}$ **11.** (a) 6, (b) 1, (c) 41
12. -12 **13.** $\frac{2(a+1)}{a-4}$ **14.** $\frac{3}{4}$ **15.** $\frac{x+1}{x-3}$
16. $\frac{2x^2 - 3x + 1}{x^2 + x - 42}$ **17.** $\frac{12p - 3}{(p-9)(p+2)(p-2)}$ **18.** $\frac{a+5}{a-6}$
19. $\left\{-\frac{1}{2}, -3\right\}$ **20.** $\{x|0 \le x \le 5\} = [0,5]$
21. $\left\{x\middle|x < \frac{3}{2} \text{ or } x > \frac{11}{2}\right\} = \left(-\infty, \frac{3}{2}\right) \cup \left(\frac{11}{2}, \infty\right)$ **22.** $\{-7\}$
23. $\{x|x \ge -33\} = [-33, \infty)$ **24.** $\left\{-\frac{5}{3}\right\}$ **25.** $w = \frac{P - 2\ell}{2}$
26. $\frac{10y - 4}{4y + 5}$ **27.** $6\sqrt{3} - 3$ **28.** $16\sqrt{3}$ **29.** $-3\sqrt[3]{3}$ **30.** 13
31. $47 - 12\sqrt{15}$ **32.** $\frac{4\sqrt{5}}{5}$ **33.** $2\sqrt{6} - 5$ **34.** $\frac{-5 + 2i}{29}$ or $\frac{-5}{29} + \frac{2}{29}i$ **35.** $\{14, 1\}$ **36.** $\left\{\frac{1 + i\sqrt{59}}{10}, \frac{1 - i\sqrt{59}}{10}\right\}$
37. $\left\{\frac{1 - i\sqrt{35}}{6}, \frac{1 + i\sqrt{35}}{6}\right\}$ **38.** $\{z|-1 \le z \le 3\} = [-1,3]$
39. $\{i\sqrt{5}, -i\sqrt{5}, \sqrt{10}, -\sqrt{10}\}$ **40.** $\{y|-7 < y < 10\} = (-7, 10)$
41. $4x^4 - 4x^3 + x^2 - 1 + \frac{2}{x+1}$

Chapter 7

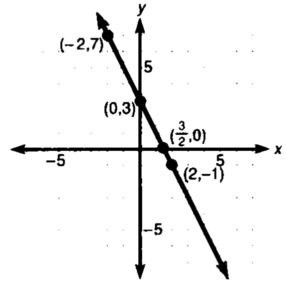
Exercise 7-1

Answers to odd-numbered problems

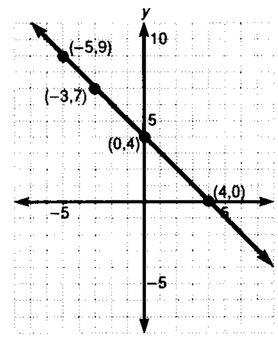
1. (2,4); quadrant I (see graph)
3. $(-4,3)$; quadrant II (see graph)
5. $(-1,-3)$; quadrant III (see graph)
7. (4,0); quadrantal (see graph)

9. $(0,-1)$; quadrantal (see graph)
11. $\left(\frac{1}{2}, 3\right)$; quadrant I (see graph)
13. $\left(-\frac{7}{2}, -\frac{5}{2}\right)$; quadrant III (see graph)

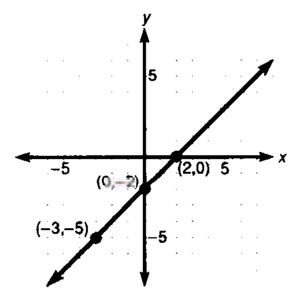
15. $(-2,7),(0,3),(2,-1),\left(\frac{3}{2},0\right)$;
$y = -2x + 3$



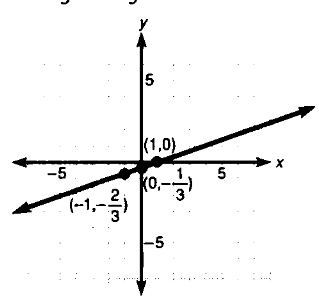
17. $(-5,9),(-3,7),(0,4),(4,0)$;
$y = -x + 4$



19. $(-3,-5),(0,-2),(2,0),(2,0)$;
$y = x - 2$

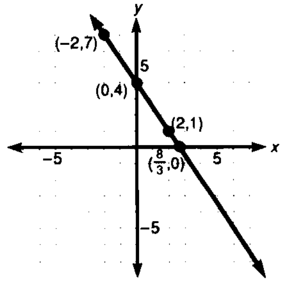


21. $\left(-1,-\frac{2}{3}\right),\left(0,-\frac{1}{3}\right),(1,0),(1,0)$;
$y = \frac{1}{3}x - \frac{1}{3}$

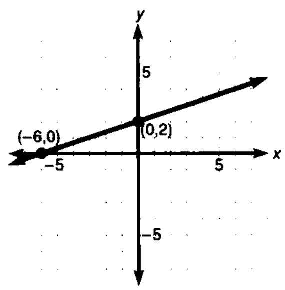


23. $(-2,7),(0,4),(2,1),\left(\frac{8}{3},0\right)$;

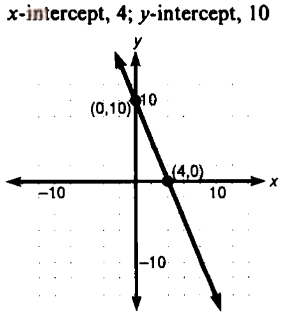
$y = -\frac{3}{2}x + 4$



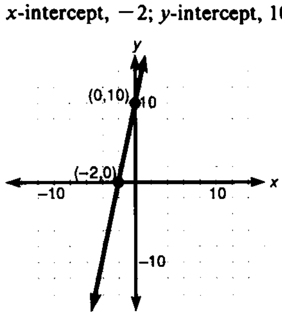
25. x-intercept, -6; y-intercept, 2



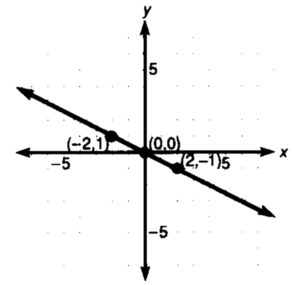
27. x-intercept, 4; y-intercept, 10



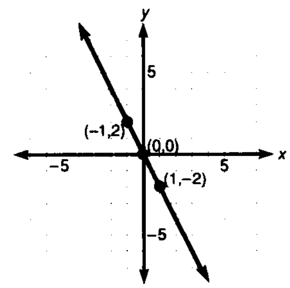
29. x-intercept, -2; y-intercept, 10



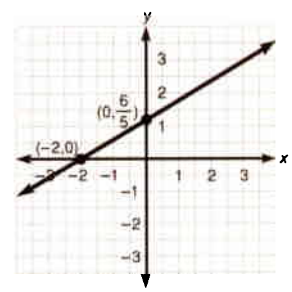
31. x-intercept, 0; y-intercept, 0



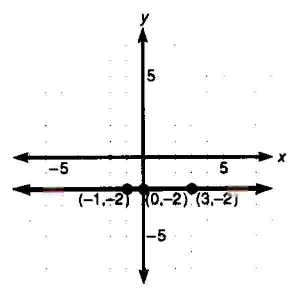
33. x-intercept, 0; y-intercept, 0



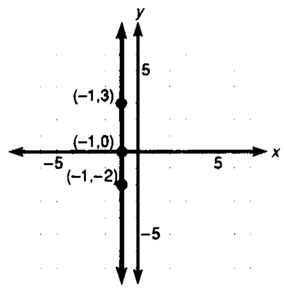
35. x-intercept, -2; y-intercept, $\frac{6}{5}$



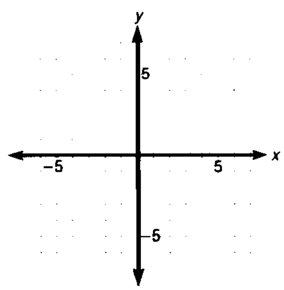
37. no x-intercept; y-intercept, -2



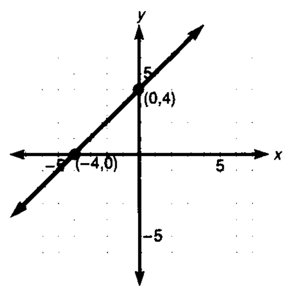
39. x-intercept, -1; no y-intercept



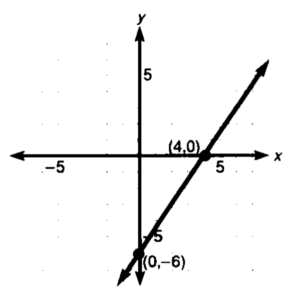
41. x-intercept, 0; all points on y-axis are y-intercepts



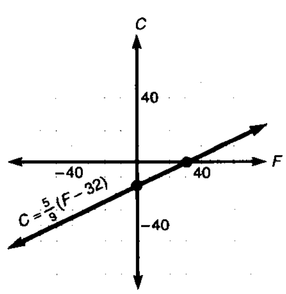
43. $y = x + 4$



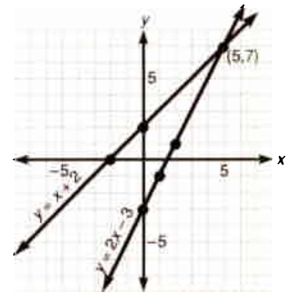
45. $3x - 2y = 12$



47. $C = \frac{5}{9}(F - 32)$



49. (5,7)



Solutions to trial exercise problems

18. $x + 2y = 4$

Solving for y, $y = \dfrac{4 - x}{2}$.

When $x = -2$, $y = \dfrac{4 - (-2)}{2} = 3$

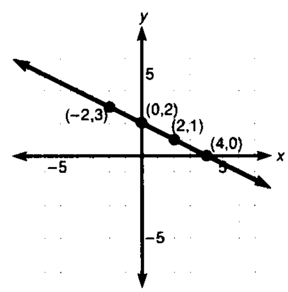
$x = 0$, $y = \dfrac{4 - 0}{2} = 2$

$x = 2$, $y = \dfrac{4 - 2}{2} = 1$

$y = 0$; $x + 2(0) = 4$

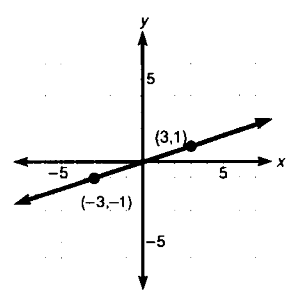
$x = 4$

$(-2,3),(0,2),(2,1),(4,0)$

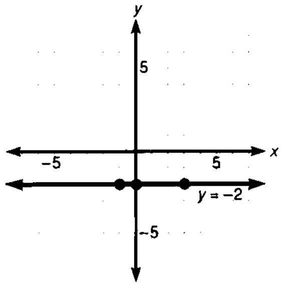


30. $x = 3y$

When $x = 0$, $3y = 0$, $y = 0$,
the x- and y-intercepts are 0.
Choose second point (3,1).

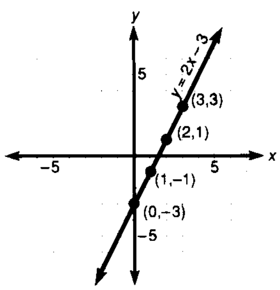


37. $y = -2$
There is no x-intercept
and the y-intercept is -2.



44. Two times x is $2x$, less 3 is $2x - 3$, is equal to y is $y = 2x - 3$.

x	-1	0	1	2	3
y	-5	-3	-1	1	3



Review exercises

1. $11 + 10i$ **2.** $17 + 0i$ **3.** $22 - 6i$ **4.** $\dfrac{4 + 3i}{5}$
5. $w = \dfrac{P - 2\ell}{2}$ **6.** $x^3 + x^2 + x + 1$ **7.** -1

Exercise 7–2

Answers to odd-numbered problems

1. $d = \sqrt{41}$ units; $m = \dfrac{5}{4}$; midpoint, $\left(4, \dfrac{9}{2}\right)$

3. $d = \sqrt{29}$ units; $m = \dfrac{5}{2}$; midpoint, $\left(-2, \dfrac{5}{2}\right)$

5. $d = 5$ units; undefined slope; midpoint, $\left(-1, \dfrac{13}{2}\right)$

7. $d = 7$ units; $m = 0$; midpoint, $\left(-\dfrac{1}{2}, 6\right)$

9. $d = \sqrt{2}$ units; $m = -1$; midpoint, $\left(-\dfrac{7}{2}, -\dfrac{1}{2}\right)$

11. $d = \sqrt{170}$ units; $m = -\dfrac{7}{11}$

13. $d = \sqrt{85}$ units; $m = -\dfrac{9}{2}$

15. $d = 10$ units; undefined slope

17. $m_1 = -3$, $m_2 = -3$, $m_1 = m_2$; parallel

19. $m_1 = -2$, $m_2 = -\dfrac{7}{11}$, $m_1 \neq m_2$; not parallel

21. $m_1 = \dfrac{1}{8}$, $m_2 = -8$, $m_1m_2 = -1$; perpendicular

23. $m_1 = 1$, $m_2 = -1$, $m_1m_2 = -1$; perpendicular

25. $m_1 = -2$, $m_2 = -2$, $m_1 = m_2$; parallel

27. $m_1 = \dfrac{3}{4}$, $m_2 = -\dfrac{4}{3}$, $m_1m_2 = -1$; perpendicular

29. $m_1 = -\dfrac{1}{4}$, $m_2 = \dfrac{2}{5}$, $m_1 \neq m_2$, $m_1m_2 \neq -1$; neither

31. $m_1 = \dfrac{1}{2}$, $m_2 = -2$, $m_1m_2 = -1$; perpendicular

33. $m_1 = \dfrac{3}{7}$, $m_2 = -\dfrac{5}{3}$, $m_1 \neq m_2$, $m_1m_2 \neq -1$; neither

35. $m_1 = \dfrac{4}{3}$, $m_2 = \dfrac{3}{4}$; neither

37. $m_1 = \dfrac{3}{4}$, $m_2 = -\dfrac{5}{3}$; neither

39. Pitch is $\dfrac{3}{5}$.

41. $m = \dfrac{2}{3}$

43. $m = \dfrac{1{,}000}{7}$

45. $m = \dfrac{15}{2}$

47. $m = -\dfrac{7}{3}$

49. Slopes of opposite sides are 2 and 0; opposite sides have lengths 4 and $\sqrt{5}$; $p = 8 + 2\sqrt{5}$ units. **51.** Slopes of two sides are $m_1 = 0$ and m_2 is undefined thus perpendicular; $6^2 + 5^2 = (\sqrt{61})^2$; $36 + 25 = 61; 61 = 61$. **53.** Two sides have slope $m_1 = m_2 = 0$, so are parallel, while the other sides have unequal slopes.
55. a. The slope using any pair of points is $\dfrac{3}{2}$. **b.** Distances between points are $\sqrt{13}$, $\sqrt{52}$, and $\sqrt{117}$; $\sqrt{13} + \sqrt{52} = \sqrt{117}$; $\sqrt{13} + 2\sqrt{13} = 3\sqrt{13}$; $3\sqrt{13} = 3\sqrt{13}$. **57.** $y = -3$ or -11
59. $(-2,12)$

Solutions to trial exercise problems

7. (3,6) and (−4,6); midpoint, $\left(\frac{3+(-4)}{2}, \frac{6+6}{2}\right) = \left(-\frac{1}{2}, 6\right)$

distance $= \sqrt{[3-(-4)]^2 + (6-6)^2}$
$= \sqrt{7^2}$
$= 7$

$m = \frac{6-6}{3-(-4)} = \frac{0}{7} = 0$

14. (0,8) and (0,−1)

distance $= \sqrt{(0-0)^2 + [8-(-1)]^2}$
$= \sqrt{0+9^2}$
$= \sqrt{81}$
$= 9$

$m = \frac{8-(-1)}{0-0} = \frac{9}{0}$ undefined

18. $m_1 = \frac{1-2}{5-(-4)} = \frac{-1}{9} = -\frac{1}{9}$

$m_2 = \frac{-3-1}{4-2} = \frac{-4}{2} = -2$ The lines are *not* parallel.

22. $m_1 = \frac{1-1}{1-4} = \frac{0}{-3} = 0$

$m_2 = \frac{2-(-3)}{-2-3} = \frac{5}{-5} = -1$

Since $m_1 \cdot m_2 = 0 \cdot -1 = 0$, the lines are not perpendicular.

31. $2y - x = 1$

Using $\left(0, \frac{1}{2}\right)$ and $(-1,0)$,

$m_1 = \frac{\frac{1}{2} - 0}{0-(-1)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$6x + 3y = 0$

Using (0,0) and (1,−2)

$m_2 = \frac{0-(-2)}{0-1} = \frac{2}{-1} = -2.$

Since $m_1 \cdot m_2 = \frac{1}{2} \cdot -2 = -1$, the lines are perpendicular.

39. $m = \text{pitch} = \frac{9 \text{ ft}}{15 \text{ ft}} = \frac{3}{5}$

45. Using (2,10) and (4,25), $m = \frac{10-25}{2-4} = \frac{-15}{-2} = \frac{15}{2}$.

51. Using (4,2) and (4,−3),

$m = \frac{2-(-3)}{4-4} = \frac{5}{0}$ (undefined).

(vertical line)

Using (−2,−3) and (4,−3),

$m = \frac{-3-(-3)}{-2-4} = \frac{-3+3}{-6} = \frac{0}{-6} = 0.$

(horizontal line)

The two lines are perpendicular so the triangle has one right angle and is a right triangle.

Distance from (4,2) to (−2,−3)
$= \sqrt{[4-(-2)]^2 + [2-(-3)]^2} = \sqrt{6^2+5^2}$
$= \sqrt{36+25} = \sqrt{61}$

Distance from (4,2) to (4,−3)
$= \sqrt{(4-4)^2 + [2-(-3)]^2} = \sqrt{0^2+5^2}$
$= \sqrt{25} = 5$

Distance from (−2,−3) to (4,−3)
$= \sqrt{(-2-4)^2 + [-3-(-3)]^2}$
$= \sqrt{(-6)^2 + 0^2} = \sqrt{36} = 6$

Now $5^2 + 6^2 = (\sqrt{61})^2$
$25 + 36 = 61$
$61 = 61$

56. Let x be the abscissa. Then using $(x,-6)$ and (4,5),

$5\sqrt{5} = \sqrt{(x-4)^2 + (-6-5)^2}$
$5\sqrt{5} = \sqrt{x^2 - 8x + 16 + 121}$
$(5\sqrt{5})^2 = (\sqrt{x^2 - 8x + 137})^2$
$125 = x^2 - 8x + 137$
$0 = x^2 - 8x + 12$
$0 = (x-6)(x-2)$, so $x = 6$ or $x = 2$.

Thus the abscissa is 6 or 2.

58. Let x be the first component of the endpoint.
Let y be the second component of the endpoint.

Then $\frac{x+(-2)}{2} = 2$
$\frac{x-2}{2} = 2$
$x - 2 = 4$
$x = 6$

$\frac{y+3}{2} = -3$
$y + 3 = -6$
$y = -9$

The other endpoint is (6,−9).

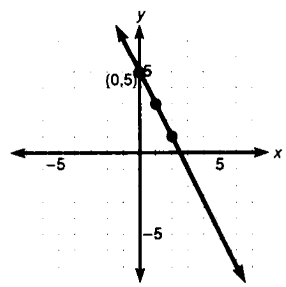
Review exercises

1. $y = \frac{-3x+4}{2}$ **2.** $y = \frac{3x+8}{4}$ **3.** $y < \frac{-x+8}{4}$
4. $y \le \frac{x-4}{2}$ **5.** $\{-36\}$ **6.** $\left\{-\frac{2}{5}\right\}$

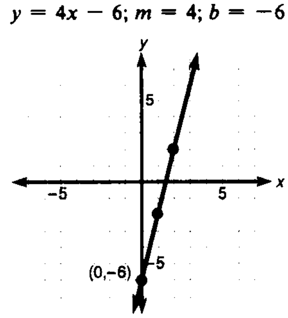
Exercise 7-3

Answers to odd-numbered problems

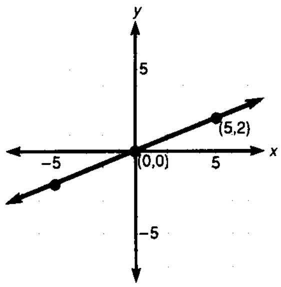
1. $x - 2y = -5$ **3.** $5x - y = -7$ **5.** $5x + 6y = 0$
7. $x - y = 3$ **9.** $y = 4$ **11.** $x = -5$ **13.** $x = 5$
15. $x = 0$ **17.** $y = -7$ **19.** $5x - 2y = 3$ **21.** $10x - 3y = 2$
23. $2x + y = 8$ **25.** $y = 4$ **27.** $x + y = 0$ **29.** $x = 4$
31. $y = -2x + 5$; $m = -2$; $b = 5$



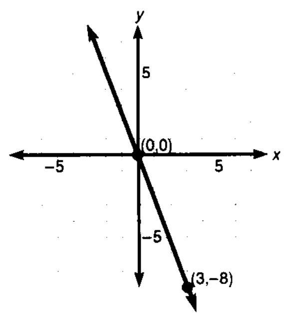
33. $y = 4x - 6$; $m = 4$; $b = -6$



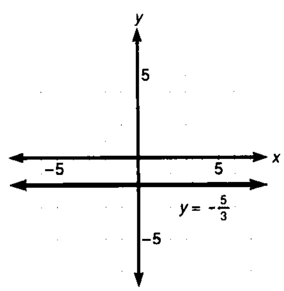
35. $y = \frac{2}{5}x$; $m = \frac{2}{5}$; $b = 0$



37. $y = -\frac{8}{3}x$; $m = -\frac{8}{3}$; $b = 0$



39. $y = -\frac{5}{3}$; $m = 0$; $b = -\frac{5}{3}$



41. $3x + y = 5$ **43.** $3x - 2y = -7$ **45.** $2x + 9y = 0$
47. $x = 4$ **49.** $y = 1$ **51.** neither **53.** perpendicular
55. parallel **57.** $y - b = \frac{b - 0}{0 - a}(x - 0)$; $y - b = -\frac{b}{a}x$;
$ay - ab = -bx$; $bx + ay = ab$; $\frac{x}{a} + \frac{y}{b} = 1$

59.
$$\begin{aligned} ax + by &= c \\ by &= -ax + c \\ y &= -\frac{a}{b}x + \frac{c}{b}; \end{aligned}$$

the slope is $-\frac{a}{b}$; the y-intercept is $\frac{c}{b}$ **61.** $m = \frac{b}{a}$

63. $250x - 3y = 550$ **65.** $140,000

Solutions to trial exercise problems

3. $m = 5$ and $(x_1,y_1) = (0,7)$
$y - 7 = 5(x - 0)$
$y - 7 = 5x$
$5x - y = -7$

6. Using $y = mx + b$, $m = -6$ and $b = 2$, $y = -6x + 2$
$6x + y = 2$.

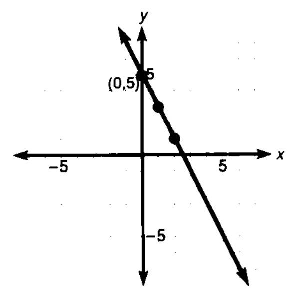
8. Horizontal line has slope 0, then $y - (-3) = 0(x - 5)$
$y + 3 = 0$; $y = -3$.

11. Having undefined slope, the line must be vertical and passing through $(-5,6)$, the *first component* of every point is -5. So $x = -5$ is the equation.

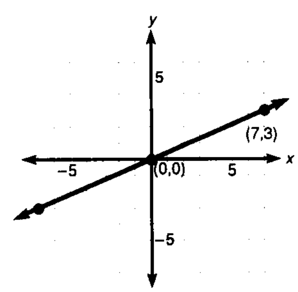
22. $(5,0)$ and $(-2,-3)$
Now $m = \frac{0 - (-3)}{5 - (-2)} = \frac{3}{5 + 2} = \frac{3}{7}$
Using point $(5,0)$ and the point-slope form,
$$y - 0 = \frac{3}{7}(x - 5)$$
$$y = \frac{3}{7}(x - 5)$$
$7y = 3x - 15$
$3x - 7y = 15$.

29. $(4,-3)$ and $(4,-7)$
$m = \frac{-3 - (-7)}{4 - 4} = \frac{4}{0}$ = undefined.
The slope is undefined, so the line is vertical and passes through $x = 4$. Thus the equation is $x = 4$.

31. $2x + y = 5$
$y = -2x + 5$
$m = -2$; $b = 5$



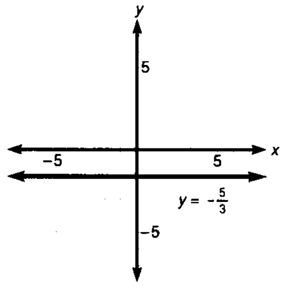
34. $3x - 7y = 0$
$y = \frac{3}{7}x + 0$
$m = \frac{3}{7}$; $b = 0$



39. $3y + 5 = 0$

$$y = -\frac{5}{3} = 0x - \frac{5}{3}$$

$m = 0;\ b = -\dfrac{5}{3}$



41. $3x + y = 6$

$y = -3x + 6$

So $m = -3$. Using $(1,2)$,

$y - 2 = -3(x - 1)$

$y - 2 = -3x + 3$

$3x + y = 5.$

49. We want the line through the midpoint that is perpendicular to the line having midpoint

$$\left(\frac{3+3}{2}, \frac{-4+6}{2}\right) = \left(\frac{6}{2}, \frac{2}{3}\right) = (3,1).$$

Now $m = \dfrac{-4-6}{3-3} = \dfrac{-10}{0}$ (undefined)

So, the line we want has slope $m = 0$ and using

$y - y_1 = m(x - x)$

$y - 1 = 0(x - 3)$

$y - 1 = 0$

$y = 1.$

The equation of the line is $y = 1$.

52. $2y - 5x = -3$

$2y = 5x - 3$

$$y = \frac{5}{2}x - \frac{3}{2}$$

So $m_1 = \dfrac{5}{2}$

$y + 3x = 4$

$y = -3x + 4$

So $m_2 = -3$.

The lines are neither parallel nor perpendicular since

$\dfrac{5}{2} \neq -3$ and $\dfrac{5}{2} \cdot -3 \neq -1.$

56. (a) $(3,2)$ and $(4,1)$

$$y - 2 = \frac{1-2}{4-3}(x - 3)$$

$$y - 2 = \frac{-1}{1}(x - 3)$$

$y - 2 = -1(x - 3)$

$y - 2 = -x + 3$

$x + y = 5$

58. (a) $4x + 3y = 12$

Divide each member by 12.

$$\frac{4x}{12} + \frac{3y}{12} = \frac{12}{12}$$

$$\frac{x}{3} + \frac{y}{4} = 1$$

The x-intercept is 3 and the y-intercept is 4.

(d) $3x - 5y = 6$

Divide each member by 6.

$$\frac{3x}{6} - \frac{5y}{6} = \frac{6}{6}$$

$$\frac{x}{2} + \frac{y}{-\frac{6}{5}} = 1$$

The x-intercept is 2 and the y-intercept is $-\dfrac{6}{5}$

62. $(300,150)$ and $(600,250)$

$$y - 150 = \frac{250 - 150}{600 - 300}(x - 300)$$

$$y - 150 = \frac{100}{300}(x - 300)$$

$$y - 150 = \frac{1}{3}(x - 300)$$

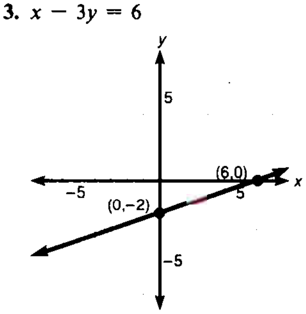
$3y - 450 = x - 300$

$x - 3y = -150$

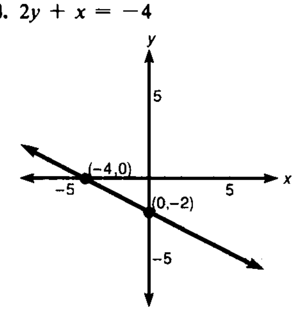
Review exercises

1. $\left\{x \middle| x \leq \frac{1}{2}\right\} = \left(-\infty, \frac{1}{2}\right]$ **2.** $y > \dfrac{3x+6}{4}$

3. $x - 3y = 6$



4. $2y + x = -4$

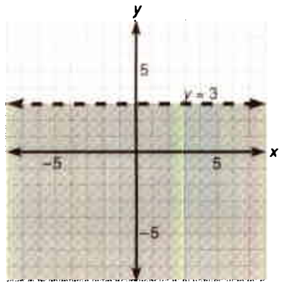


5. $\left\{\dfrac{3 - \sqrt{41}}{4}, \dfrac{3 + \sqrt{41}}{4}\right\}$ **6.** $\{6\}$; 1 is extraneous

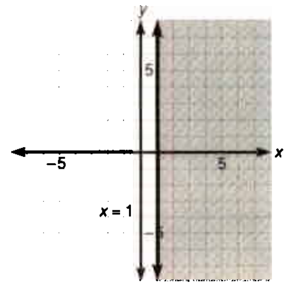
Exercise 7–4

Answers to odd-numbered problems

1.



3.



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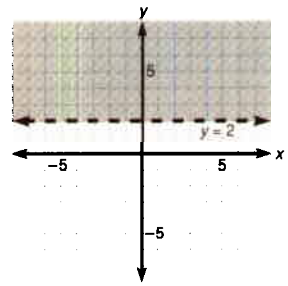

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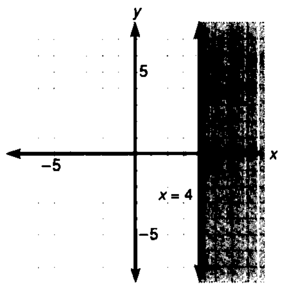

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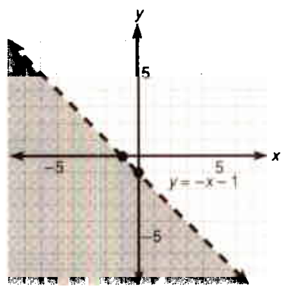

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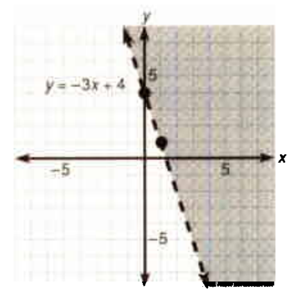

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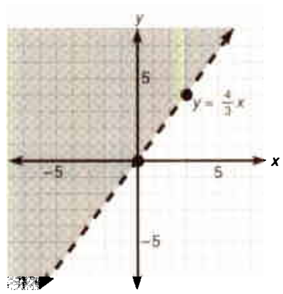

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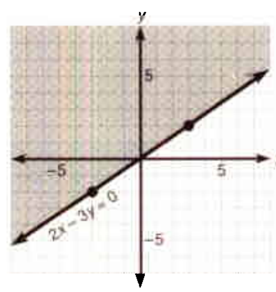

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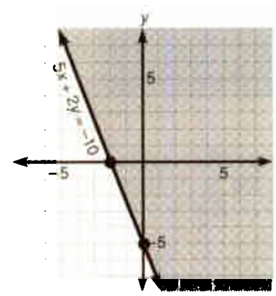

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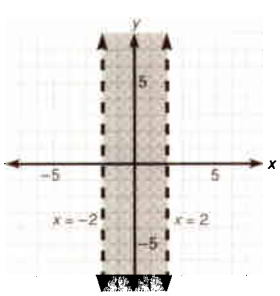

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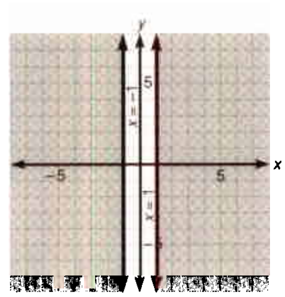

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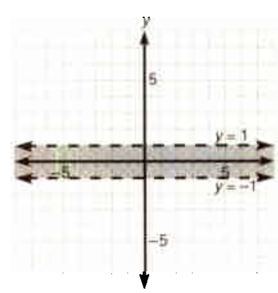

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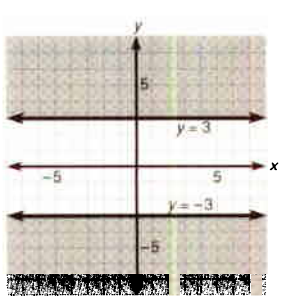

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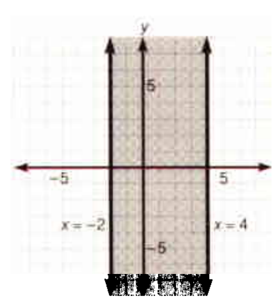

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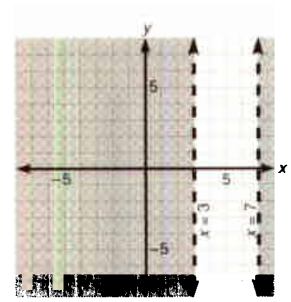

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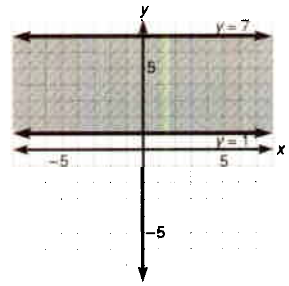
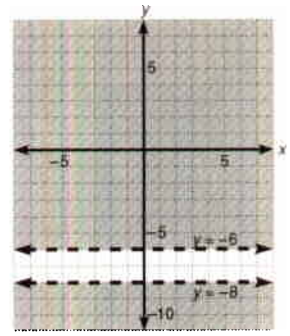


33.



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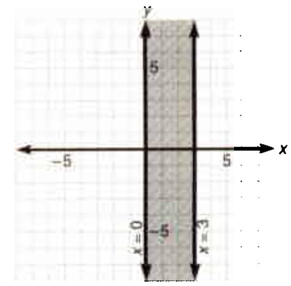
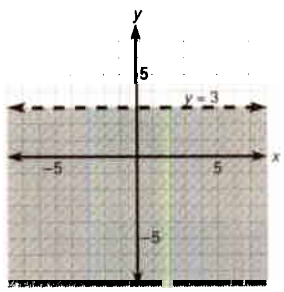
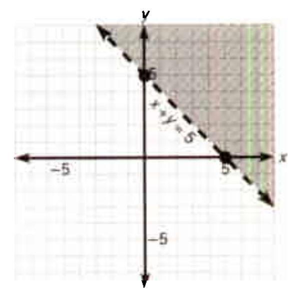


Solutions to trial exercise problems

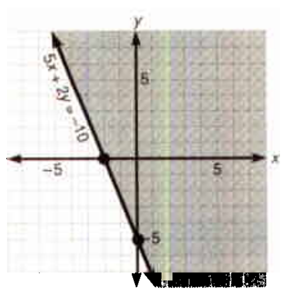
1. $y < 3$
Since every point having $y < 3$ lies below the line $y = 3$, we dash the horizontal line $y = 3$ and shade the plane below this horizontal line.



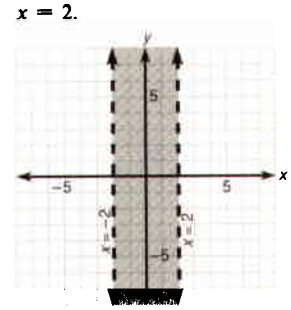
8. $x + y > 5$
Graph the line $x + y = 5$ (dashed) and since for (0,0), $0 + 0 > 5$ is false, shade *above* this line.



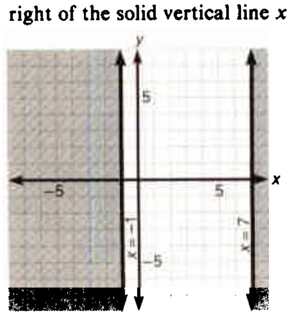
17. $5x + 2y \geq -10$
Graph the line $5x + 2y = -10$ (make this line *solid*). Since for (0,0), $5(0) + 2(0) \geq -10$ is true, shade the plane to the right of this line.



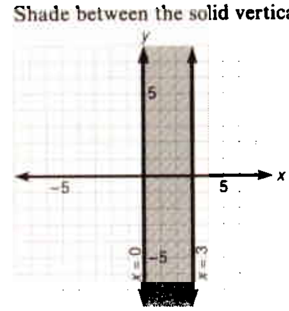
19. $|x| < 2$, $-2 < x < 2$
Shade the plane between the dashed vertical lines, $x = -2$ and $x = 2$.



30. $|3 - x| \geq 4$
$3 - x \geq 4$ or $3 - x \leq -4$
$x \leq -1$ or $x \geq 7$
Shade to the left of the solid vertical line $x = -1$ and to the right of the solid vertical line $x = 7$.

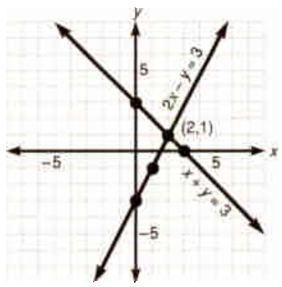


35. $|2x - 3| \leq 3$
$-3 \leq 2x - 3 \leq 3$
$0 \leq 2x \leq 6$
$0 \leq x \leq 3$
Shade between the solid vertical lines $x = 0$ (y-axis) and $x = 3$.

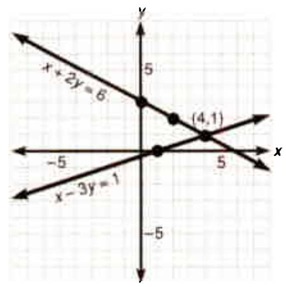


Review exercises

1.



2.



3. $P(-1) = 7$
$P(2) = 13$

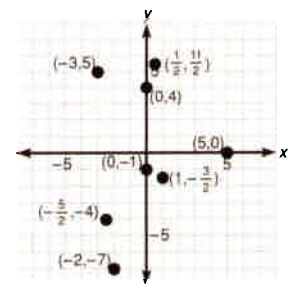
4. -40

5. $\dfrac{3x^2 - 11x + 4}{(x-3)(x+3)}$

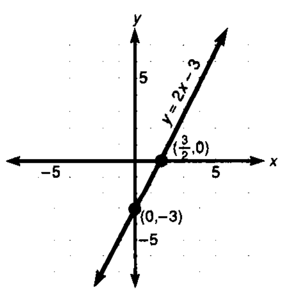
6. $\dfrac{x^2}{12y}$

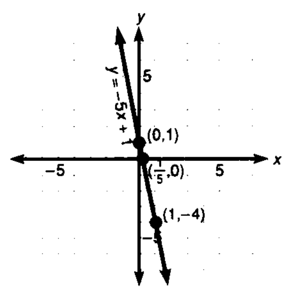
Chapter 7 review

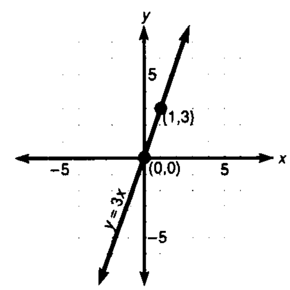
1. $(-3,5)$; quadrant II (see graph)
2. $(-2,-7)$; quadrant III (see graph)
3. $(0,-1)$; quadrantal (see graph)
4. $(5,0)$; quadrantal (see graph)
5. $\left(1,-\dfrac{3}{2}\right)$; quadrant IV (see graph)
6. $\left(\dfrac{1}{2},\dfrac{11}{2}\right)$; quadrant I (see graph)
7. $(0,4)$; quadrantal (see graph)
8. $\left(-\dfrac{5}{2},-4\right)$; quadrant III (see graph)



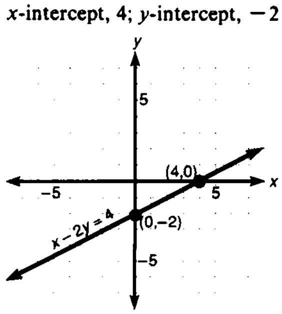
9. x-intercept, 6; y-intercept, -2
10. x-intercept, 4; y-intercept, -8
11. x-intercept, 2; y-intercept, -5
12. x-intercept, -6; no y-intercept
13. no x-intercept; y-intercept, 5
14. x-intercept, $\dfrac{7}{2}$; y-intercept, $\dfrac{7}{4}$
15. x-intercept, $\dfrac{3}{2}$; y-intercept, -3
16. x-intercept, $\dfrac{1}{5}$; y-intercept, 1
17. x-intercept, 0; y-intercept, 0



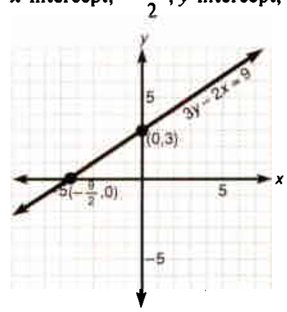




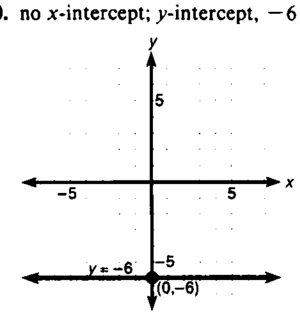
18. x-intercept, 4; y-intercept, -2



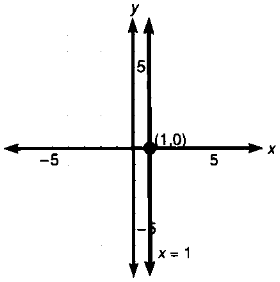
19. x-intercept, $-\dfrac{9}{2}$; y-intercept, 3



20. no x-intercept; y-intercept, -6



21. x-intercept, 1; no y-intercept



22. $\sqrt{13}$ units midpoint, $\left(\frac{3}{2},3\right)$; $m = -\frac{2}{3}$

23. 3 units; midpoint, $\left(-2,\frac{5}{2}\right)$; m is undefined

24. 4 units midpoint, $(-1,5)$; $m = 0$

25. neither; $m_1 = \frac{1}{4}$, $m_2 = \frac{-3}{2}$

26. perpendicular; $m_1 = 2$, $m_2 = -\frac{1}{2}$

27. parallel; $m_1 = \frac{4}{7} = m_2$

28. neither; $m_1 = -\frac{1}{3}$, $m_2 = \frac{5}{9}$

29. parallel; $m_1 = -2 = m_2$

30. neither; $m_1 = \frac{1}{3}$, $m_2 = -\frac{2}{3}$

31. perpendicular; $m_1 = \frac{2}{5}$, $m_2 = -\frac{5}{2}$

32. perpendicular; $m_1 = \frac{3}{2}$, $m_2 = -\frac{2}{3}$

33. $m = \frac{18}{13}$

34. $7^2 + 3^2 = (\sqrt{58})^2$; $49 + 9 = 58$; $58 = 58$

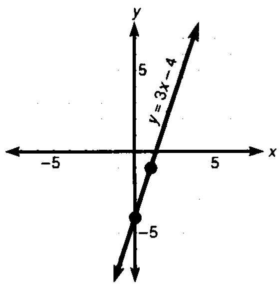
35. $2x - 3y = -17$

36. $y = 4$

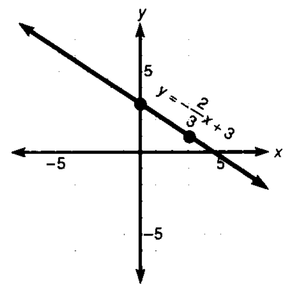
37. $x = 1$

38. $8x - y = 21$

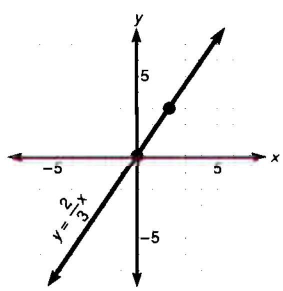
39. $y = 3x - 4$; $m = 3$; y-intercept $= -4$



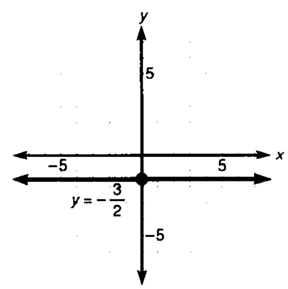
40. $y = \frac{-2}{3}x + 3$; $m = \frac{-2}{3}$; y-intercept $= 3$



41. $y = \frac{3}{2}x$; $m = \frac{3}{2}$; y-intercept $= 0$



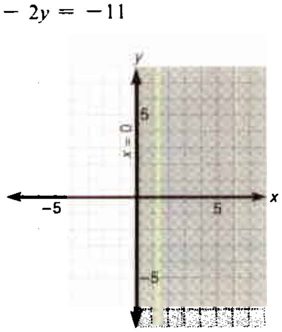
42. $y = \frac{-3}{2}$; $m = 0$; y-intercept $= \frac{-3}{2}$



43. $x - 2y = -11$

44. $5x - 4y = -12$

45.



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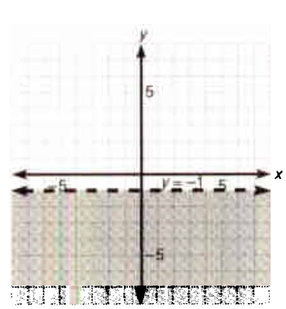

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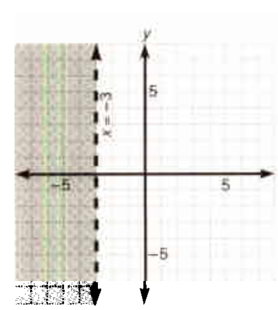

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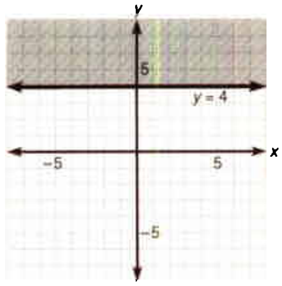

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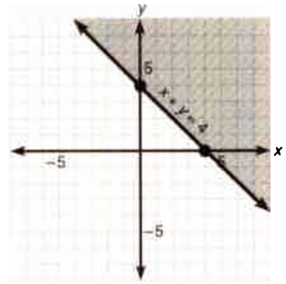

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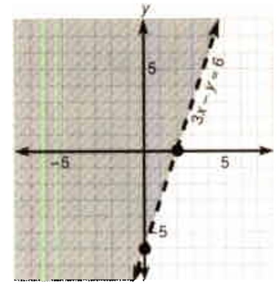

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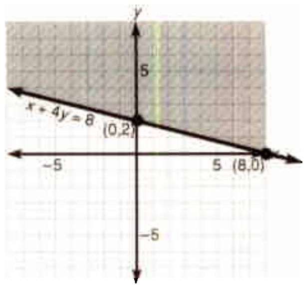

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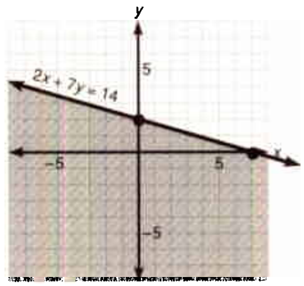

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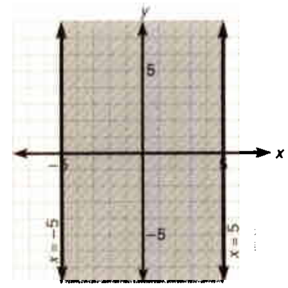

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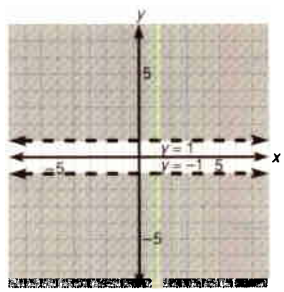

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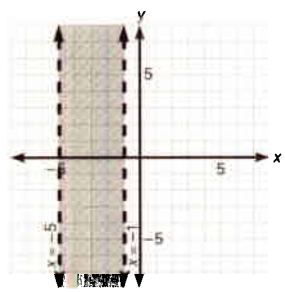

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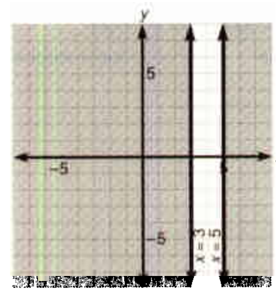

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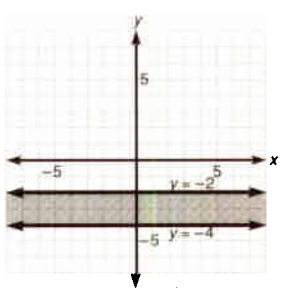

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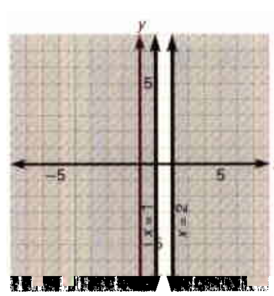


Chapter 7 cumulative test

1. $-18a^3b^5$ **2.** $\dfrac{y^6}{x^5}$ **3.** $\dfrac{a^6c^2}{16b^4}$ **4.** $9y^2 - 3y - 3$

5. $-3x^2 - 8xy + 3y^2$ **6.** $12x^3y - 18xy^3 + 6x^3y^2 - 6x^4y^4$

7. $3(a + 2b)(a - 2b)$ **8.** $(8x + 1)(x - 7)$

9. $(2x + 3y)(4x^2 - 6xy + 9y^2)$ **10.** $(4x - 5y)^2$ **11.** $\left\{-\dfrac{45}{4}\right\}$

12. $\{1, -4\}$ **13.** $\left\{y \,\middle|\, -\dfrac{3}{2} < y < \dfrac{5}{2}\right\} = \left(-\dfrac{3}{2}, \dfrac{5}{2}\right)$

14. $\{y|y \leq -4 \text{ or } y \geq -2\} = (-\infty,-4] \cup [-2,\infty)$

15. $\left\{0,\frac{1}{4}\right\}$ **16.** $\{9,-2\}$ **17.** $\left\{\frac{3}{2},-1\right\}$

18. $\left\{x\middle|-\frac{1}{3} \leq x < 3\right\} = \left[-\frac{1}{3},3\right)$ **19.** $\left\{\frac{3+\sqrt{65}}{4},\frac{3-\sqrt{65}}{4}\right\}$

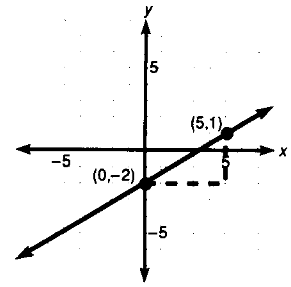
20. $\frac{1}{y-5}$ **21.** $\frac{-3y^2+46y-21}{2y(y+7)(y-7)}$ **22.** 1 **23.** $\left\{\frac{11}{10}\right\}$

24. a. $P(-3) = 0$ **b.** $x + 3$ is a factor of $3x^3 + 8x^2 - 7x = 12$

25. $6\sqrt{2}$ **26.** $21 + 8\sqrt{5}$ **27.** $\frac{3\sqrt{5}}{5}$ **28.** $6 + 3\sqrt{3}$ **29.** 13

30. $2 - 7i$ **31.** $\emptyset$; 7 is extraneous **32.** $7x - y = 10$

33. $2x - 3y = -8$ **34.** $x = 5$ **35.** $m = \frac{3}{5}, b = -2$



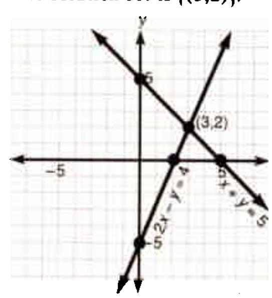
36. perpendicular **37.** $d = \sqrt{65}$; midpoint, $\left(\frac{3}{2},1\right)$

Chapter 8

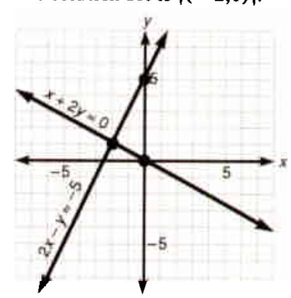
Exercise 8-1

Answers to odd-numbered problems

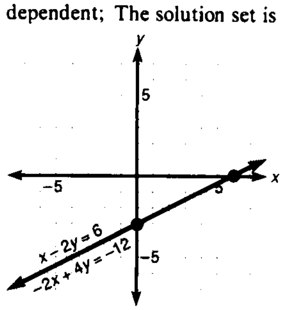
1. yes **3.** yes **5.** $(-1,2)$ not a solution **7.** The solution set is $\{(3,2)\}$.



9. The solution set is $\{(-2,1)\}$.



11. dependent; The solution set is $\{(x,y)|x - 2y = 6\}$.



13. $\{(-2,-5)\}$

15. $\{(-2,3)\}$ **17.** $\{(2,3)\}$ **19.** $\{(3,1)\}$ **21.** $\left\{\left(\frac{3}{2},1\right)\right\}$

23. $\{(-1,-4)\}$ **25.** $\{(x,y)|3x + y = 2\}$; dependent

27. $\emptyset$; inconsistent **29.** $\left\{\left(\frac{5}{9},\frac{10}{9}\right)\right\}$ **31.** $\left\{\left(\frac{5}{3},\frac{1}{2}\right)\right\}$

33. $\left\{\left(\frac{3}{5},0\right)\right\}$ **35.** $\left\{\left(-2,\frac{7}{2}\right)\right\}$ **37.** $\{(3,-2)\}$

39. $\{(7,-4)\}$ **41.** $\left\{\left(\frac{7}{2},-\frac{3}{2}\right)\right\}$ **43.** $\{(1,-3)\}$ **45.** $\{(-1,-4)\}$

47. $\{(4,12)\}$ **49.** $\{(-2,-5)\}$ **51.** $\{(6,6)\}$ **53.** $\{(-1,0)\}$

55. $\emptyset$; inconsistent **57.** $\{(x,y)|2x - y = 7\}$; dependent

59. $\left\{\left(\frac{5}{2},2\right)\right\}$ **61.** $\left\{\left(\frac{3}{8},\frac{33}{8}\right)\right\}$ **63.** $\left\{\left(-\frac{12}{11},-\frac{63}{11}\right)\right\}$

65. $\left\{\left(-1,\frac{1}{4}\right)\right\}$ **67.** $\left\{\frac{7}{5},-\frac{7}{4}\right\}$ **69.** $x + y = 502$

71. $x = y + 6$ or $y = x + 6$ **73.** $y = 3x + 4$ or $x = 3y + 4$

75. $x - y = 33$

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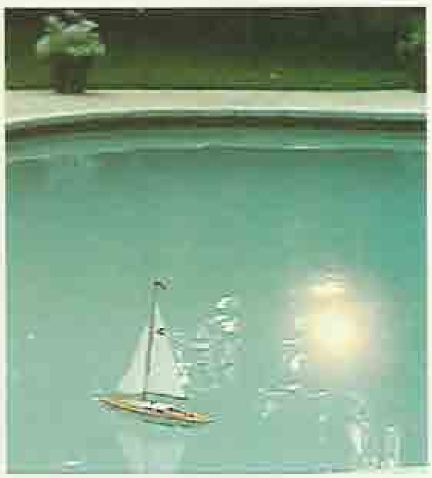

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